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AMERICAN MATHEMATICAL SOCIETY

Notices

Edited by J. H. CURTISS

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Contents

MEETINGS
Calendar of Meetings 704
Program of the January Meeting 705
Abstracts for the Meeting, pp. 771-855

PRELIMINARY ANNOUNCEMENT OF MEETING 730

ACTIVITIES OF OTHER ASSOCIATIONS 731

NEWS ITEMS AND ANNOUNCEMENTS 733

LETTERS TO THE EDITOR 750

MEMORANDA TO MEMBERS
Corporate and Institutional Members 751
Catalogue of Lecture Notes 756

NEW PUBLICATIONS 764

ABSTRACTS OF CONTRIBUTED PAPERS 771

INDEX OF ABSTRACTS, VOLUME 5 856

Please send in abstracts of papers to be presented in person well in advance of the deadline.

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MEETINGS

CALENDAR OF MEETINGS

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts *
554	February 28, 1959	New York, New York	Jan. 15
555	April 17-18, 1959	Chicago, Illinois	Mar. 4
556	April 17-18, 1959	Monterey, California	Mar. 4
557	April 23-25, 1959	New York, New York	Mar. 4
558	June 20, 1959	Eugene, Oregon	May 7
559	September 2-5, 1959 (64th Summer Meeting)	Salt Lake City, Utah	July 20
560	October 31, 1959	Cambridge, Massachusetts	Sept. 17
	November, 1959	Detroit, Michigan	
	January 27-29, 1960 (66th Annual Meeting)	Chicago, Illinois	

*The abstracts of papers to be presented at the meetings must be received in the Headquarters Offices of the Society in Providence, R. I., on or before these deadlines. The deadlines also apply to news items.

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SIXTY-FIFTH ANNUAL MEETING

University of Pennsylvania
Philadelphia, Pennsylvania
January 20-22, 1959

PROGRAM

The sixty-fifth Annual Meeting of the American Mathematical Society will be held on Tuesday, Wednesday, and Thursday, January 20-22, 1959, at the University of Pennsylvania in Philadelphia, Pennsylvania, in conjunction with meetings of the Mathematical Association of America on Thursday and Friday, January 22 and 23, the Association for Symbolic Logic on Thursday, January 22, and the Delaware Valley Section of the Society for Industrial and Applied Mathematics on Wednesday, January 21.

The thirty-second Josiah Willard Gibbs Lecture will be delivered by Professor J. M. Burgers of the University of Maryland on Tuesday at 8:00 P.M. in the auditorium of the University Museum. The title of the lecture is "On the emergence of patterns of order."

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, Professor G. D. Mostow of Johns Hopkins University will deliver an address on "Compact transformation groups" on Tuesday at 2:00 P.M., and Professor Felix Browder of Yale University will address the Society on "The spectral theory of partial differential operators" on Wednesday at 2:00 P.M. Both addresses will be given in the auditorium of the University Museum.

The award of the Bôcher Prize will be announced on Thursday at 2:00 P.M. in the auditorium of the University Museum, and the recipient will deliver an address upon his investigations. This award will be followed by the Annual Business Meeting.

There will be a banquet on Thursday evening at 7:00 P.M. at the Benjamin Franklin Hotel.

The Council of the Society will meet on Wednesday at 5:00 P.M. in the William Penn room of the Benjamin Franklin Hotel, reconvening after dinner.

There will be a meeting of the Board of Trustees at 10:00 A.M. on Thursday in Room 10 on the third floor of Houston Hall, reconvening after luncheon.

Sessions for contributed papers will be held on Tuesday, Wednesday, and Thursday at 10:00 A.M. and 3:15 P.M. Abstracts of the papers to be presented in these sessions appear on pages 788-854 of these NOTICES. There are cross references to the abstracts in the program. Thus, for example, the title of paper (1) in the program is followed by (553-103), indicating that the abstract can be found under the designation 553-103 among the published abstracts. Sessions for late papers will be held at 3:15 P.M. on Thursday. The new Associate

Secretary, Professor Everett Pitcher of Lehigh University, will be in charge of the meeting; all correspondence concerning late papers and changes in the program should be addressed to him at the Department of Mathematics, Lehigh University, Bethlehem, Pennsylvania.

The Employment Register will be maintained in the Bishop White room of Houston Hall throughout the meeting.

REGISTRATION

Registration headquarters will be in the Foyer of Houston Hall (Spruce Street, between 34th and 36th Streets). The desk will be maintained on Tuesday through Thursday from 9:00 A.M. to 5:00 P.M. and on Friday from 9:00 A.M. to 2:00 P.M. In accordance with the Society's recent decision to charge registration fees at Annual Meetings as well as at Summer Meetings, there will be a registration fee of $1.00 for each member of any participating organization and $.50 for each accompanying adult.

There will be a book exhibit in Houston Hall in the lobby of the second floor.

ROOMS AND MEALS

Accommodations will be available at hotels. The following hotels, listed in order of increasing distance from the University of Pennsylvania (the first is within walking distance), have agreed to set aside blocks of rooms for those attending the meeting:

The Penn Sherwood, 39th and Chestnut Streets, Philadelphia 1. (Rates: singles $6.75; doubles $9.00; twin bedded rooms $11.00).

Sheraton Hotel, 1725 Pennsylvania Boulevard, Philadelphia 3. (Rates: singles $8.50 - 18.50; doubles $13.00 - 15.00; twin bedded rooms $15.00 - 22.00; rollaway beds $3.00).

The Benjamin Franklin Hotel, 9th and Chestnut Streets, Philadelphia 5. (Rates: singles $6.00; doubles or twin bedrooms $10.00; rooms for three $4.50 per person).

Members should make their own reservations not later than January 5, 1959. In their letters of reservation they should refer to the meeting of the American Mathematical Society, as some of the quoted prices are special rates for members of the participating organizations only. (In the preliminary announcement of this meeting, the Hotel Normandie was also listed, but rooms are no longer available at that hotel.)

Meals can be taken at several cafeterias on the University of Pennsylvania campus. A list of restaurants close to the campus and downtown will be distributed at the registration desk.

ENTERTAINMENT AND RECREATION

There will be an official reception by the University of Pennsylvania on Wednesday from 4:00 to 6:00 P.M. in the Rotunda of the University Museum.

A social get-together, sponsored by the Delaware Valley section of the Society for Industrial and Applied Mathematics, will be held on Wednesday at 9:00 P.M. in the Rotunda of the University Museum. Tickets for $1.00 will be available at the Registration Desk.

The banquet will be held at the Benjamin Franklin Hotel at 7:00 P.M. on Thursday evening. Tickets for the banquet ($5.00 each) should be obtained at the time of registration.

Conducted tours of the University campus including the Computer Center and University Museum, and possibly of Philadelphia, will be organized in case there is interest in them. Maps of the campus, maps of Philadelphia, lists of restaurants, addresses for shopping downtown and sites of interest in Philadelphia will be available at the Registration Desk.

TRANSPORTATION

Philadelphia may be reached by train (the Pennsylvania Railroad and the Reading Line), by bus (Greyhound, Quaker Line, National Trailways), by car (Pennsylvania and New Jersey Turnpikes, U. S. Routes 1, 13, 30, 130, 309, 611), or by airplane (International Airport in South Philadelphia). From the various terminals, taxis may be taken to the hotels. Within two blocks of the railroad stations (PRR 30th Street Station and the Reading Terminal at 12th and Market Streets), there are bus lines D and 42, leading to the Benjamin Franklin Hotel and to the University of Pennsylvania. In order to reach the Penn Sherwood Hotel, one should use only the D bus from the station; it stops one block from the hotel.

All sessions of the meeting will take place at the University of Pennsylvania. Houston Hall is located on Spruce Street, between 34th and 36th Streets. The University Museum is on the southeast corner of 34th and Spruce Streets, entrance through 34th Street. The Physical Sciences Building is located on the southeast corner of 33rd and Walnut Streets. The Moore School Building is on the southwest corner of 33rd and Walnut Streets.

MAIL AND TELEGRAMS

Correspondence for those attending the meetings should be addressed in care of the American Mathematical Society, Houston Hall, University of Pennsylvania, Philadelphia 4, Pennsylvania.

Committee on Arrangements

P. A. Caris	H. M. Gehman	G. E. Schweigert
J. H. Curtiss	W. H. Gottschalk	C. T. Yang
Robert Ellis	R. D. Schafer	Emil Grosswald, Chairman

TIME TABLE
(Eastern Standard Time)

TUESDAY, January 20	American Mathematical Society
9:00 A.M. - 5:00 P.M.	REGISTRATION - FOYER, HOUSTON HALL
9:00 A.M. - 5:00 P.M.	EMPLOYMENT REGISTER - BISHOP WHITE ROOM, HOUSTON HALL
10:00 A.M.	Sessions for Contributed Papers Analysis, Physical Sciences Building, Auditorium A-1 Topology, Physical Sciences Building, Auditorium A-2 Algebra & Theory of Numbers, Physical Sciences Building, Auditorium A-4 Statistics & Probability, Moore School Building, Auditorium
2:00 P.M.	Invited Address, G. D. Mostow - University Museum, Auditorium
3:15 P.M.	Sessions for Contributed Papers Analysis, Physical Sciences Building, Auditorium A-1 Geometry, Physical Sciences Building, Auditorium A-2 Algebra & Theory of Numbers, Physical Sciences Building, Auditorium A-4 Applied Mathematics, Moore School Building, Auditorium
8:00 P.M.	Gibbs Lecture, J. M. Burgers - University Museum, Auditorium

TIME TABLE (Cont.)
(Eastern Standard Time)

WEDNESDAY, January 21	American Mathematical Society	Society for Industrial and Applied Mathematics
9:00 A.M. - 5:00 P.M.	REGISTRATION - FOYER, HOUSTON HALL	
9:00 A.M. - 5:00 P.M.	EMPLOYMENT REGISTER - BISHOP WHITE ROOM, HOUSTON HALL	
10:00 A.M.	Sessions for Contributed Papers Analysis, Physical Sciences Bldg., Auditorium A-1 Analysis, Physical Sciences Bldg., Auditorium A-2 Algebra & Theory of Numbers, Physical Sciences Bldg., Auditorium A-4 Applied Mathematics, Moore School Bldg., Auditorium	---
2:00 P.M.	Invited Address, Felix Browder - University Museum, Auditorium	---
3:15 P.M.	Sessions for Contributed Papers Analysis, Physical Sciences Bldg., Auditorium A-1 Topology, Physical Sciences Bldg., Auditorium A-2 Algebra & Theory of Numbers, Physical Sciences Bldg., Auditorium A-4 Logic & Foundations, Moore School Bldg., Auditorium	---
4:00 P.M. - 6:00 P.M.	Official reception by the University of Pennsylvania, Rotunda, University Museum	
5:00 P.M.	Council Meeting, William Penn Room, Benjamin Franklin Hotel	
8:00 P.M.	---	Lecture by John Mauchly, Physical Sci. Bldg.
9:00 P.M.	---	Social Get-Together, Rotunda, Univ. Museum

TIME TABLE (Cont.)
(Eastern Standard Time)

THURSDAY, January 22	American Mathematical Society	Association for Symbolic Logic	Mathematical Association of America
9:00 A.M. - 5:00 P.M.	REGISTRATION - FOYER, HOUSTON HALL		
9:00 A.M. - 5:00 P.M.	EMPLOYMENT REGISTER - BISHOP WHITE ROOM, HOUSTON HALL		
9:00 A.M.	---	---	First Session, University Museum, Auditorium
10:00 A.M.	Board of Trustees, Houston Hall, Third Floor, Room 10	---	---
10:00 A.M.	Sessions for Contributed Papers Analysis, Physical Sciences Bldg., Auditorium A-1 Analysis, Physical Sciences Bldg., Auditorium A-2 Geometry, Physical Sciences Bldg., Auditorium A-4 Statistics & Probability, Moore School Bldg., Audit.	---	---
10:30 A.M.	---	Invited Address, Th. Skolem Moore School Bldg., Room 212	---
	---	Session for Contributed Papers Moore School Bldg., Room 212	---
2:00 P.M.	Bôcher Prize, University Museum, Auditorium	---	---
2:00 P.M.	Annual Business Meeting, University Museum, Auditorium	---	---
2:30 P.M.	---	Invited Address, J. B. Rosser Moore School Bldg., Room 212	---

TIME TABLE (Cont.)
(Eastern Standard Time)

THURSDAY, January 22 (Continued)	American Mathematical Society	Association for Symbolic Logic	Mathematical Association of America
	---	Session for Contributed Papers Moore School Bldg., Room 212	---
3:15 P.M.	Sessions for Contributed Papers Analysis, Physical Sciences Bldg., Auditorium A-1 Applied Mathematics, Moore School Bldg., Auditorium	---	---
3:15 P.M.	Sessions for Late Papers	---	---
3:30 P.M.	---	---	Board of Governors, Poor Richard Room, Benjamin Franklin Hotel
7:00 P.M.	BANQUET, BENJAMIN FRANKLIN HOTEL		

FRIDAY, January 23	American Mathematical Society	Association for Symbolic Logic	Mathematical Association of America
9:00 A.M. - 2:00 P.M.	REGISTRATION - FOYER, HOUSTON HALL		
9:00 A.M. - 2:00 P.M.	EMPLOYMENT REGISTER - BISHOP WHITE ROOM, HOUSTON HALL		
9:00 A.M.	---	---	Second Session, University Museum, Auditorium
2:00 P.M.	---	---	Business Meeting, University Museum, Auditorium
2:00 P.M.	---	---	Third Session, University Museum, Auditorium

PROGRAM OF THE SESSIONS
(Time limit for each contributed paper, 10 minutes)

TUESDAY, 10:00 A.M.

Session on Analysis, Physical Sciences Building, Auditorium A-1

(1) Representation of martingales
Professor J. L. Snell and Professor R. E. Williamson, Dartmouth College (553-101)

(2) Explicit perturbation formulae and convergence theorems
Professor F. H. Brownell, University of Washington (553-40)

(3) On dynamical systems without improper saddle points. I
Dr. Pinchas Mendelson, Columbia University (553-132)

(4) The periodic solutions of the N-body problem
Dr. R. W. Bass, Princeton University and RIAS, Inc., Baltimore, Maryland (553-139)

(5) Existence and uniqueness of the periodic solution of an equation for autonomous oscillations
Dr. R. P. de Figueiredo, Junta de Investigacoes do Ultramer
(Introduced by Dr. S. H. Gould)

(6) The behavior of Rayleigh's equation at infinity
Professor Abolghassem Ghaffari, National Bureau of Standards, Washington, D. C. and Teheran University (553-60)

(7) Structural stability in the plane with enlarged boundary conditions
Professor M. C. Peixoto, Princeton University, and Dr. M. M. Peixoto, RIAS, Inc., Baltimore, Maryland

(8) Structurally stable differential equations
Professor L. F. Markus, University of Minnesota (553-91)

(9) On the behavior of periodic linear differential systems near resonance points
Dr. J. K. Hale, RIAS, Inc., Baltimore, Maryland

Session on Topology, Physical Sciences Building, Auditorium A-2

(10) Regular mappings whose inverses are 2-cells
Professor Mary-Elizabeth Hamstrom, Goucher College (553-24)

(11) Minimal sets under flows on tori and derived spaces
Professor R. D. Anderson, Louisiana State University (553-133)

(12) Equicontinuous and related flows
Dr. C. H. Cunkle, Cornell Aeronautical Laboratory, and Professor W. R. Utz, University of Missouri (553-72)

(13) Chromatic polynomials
Mrs. B. R. Vanderslice, University of Maryland, and Professor D. W. Hall, Harpur College

(14) A general inequality for a Lebesgue type area
Professor R. F. Williams, Purdue University (553-156)

(15) Summability in compact Abelian groups
Professor Mark Mahowald, Xavier University Cincinnati, Ohio (553-25)

(16) A theorem on equidistribution in compact groups. II
Professor G. M. Helmberg, Tulane University (553-62)

(17) Bonded groups
Mr. Hsin Chu, University of Pennsylvania (553-135)

Session on Algebra and Theory of Numbers, Physical Sciences Building, Auditorium A-4

(18) Skew matrices as square roots
Dr. R. F. Rinehart, Duke University (553-53)

(19) A graph theoretic method for the reduction of a matrix
Professor Frank Harary, University of Michigan, Princeton University and Institute for Advanced Study (553-4)

(20) Some exponential sums for matrices over a finite field. Preliminary report
Dr. J. H. Hodges, Cornell Aeronautical Laboratory, Buffalo, New York (553-47)

(21) Concerning non-invariant submodules of simple, matrix, rings. II
Professor C. C. Faith, Pennsylvania State University (553-140)

(22) Structure of cleft rings. II
Professor J. H. Walter, University of Washington

(23) A structure theory for a class of lattice-ordered rings. I. Preliminary report
Mr. D. G. Johnson, Purdue University (553-64)

(24) Modularity relations in lattices
Dr. R. J. Mihalek, Illinois Institute of Technology (553-92)

(25) Retracts in Boolean algebras
Professor Philip Dwinger, Purdue University (553-23)

(26) The free product of polyadic algebras and applications
Mr. Aubert Daigneault, Royal Military College of Canada (553-29)

Session on Statistics and Probability, Moore School Building, Auditorium

(27) Stochastic processes with analytic covariance
Dr. S. P. Lloyd, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey (553-3)

(28) On the entrance-boundary for Markov processes with discrete state space
Professor Donald Austin, University of Miami (553-13)

(29) Ergodic theorems for discrete semi-Markov processes. Preliminary report
Professor P. M. Anselone, University of Wisconsin (553-106)

(30) Recurrent Markov chains. II
Professor Steven Orey, University of Minnesota (553-147)

(31) A class of integral transforms suggested by probability theory
Professor R. K. Getoor, University of Washington (553-19)

(32) Brownian motion of rotation
Mr. C. D. Gorman, University of Washington (553-22)

(33) On a class of distribution functions where the quotient follows the Cauchy law
Dr. R. G. Laha, The Catholic University of America (553-28)
(Introduced by Professor Eugene Lukacs)

(34) General stochastic processes in models of telephone traffic and type I counters
Dr. V. E. Beneš, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey (553-77)

TUESDAY, 2:00 P.M.

Invited Address, University Museum, Auditorium
Compact transformation groups (One hour)
Professor G. D. Mostow, Johns Hopkins University

TUESDAY, 3:15 P.M.

Session on Analysis, Physical Sciences Building, Auditorium A-1

(35) Some applications of centroids to problems in numerical analysis
Dr. S. I. Askovitz, University of Pennsylvania and Albert Einstein Medical Center (553-108)

(36) On the maximum modulus of polynomials
Dr. T. J. Rivlin, Fairchild Engine and Airplane Corporation, Deer Park, Long Island, New York (553-52)

(37) A strong maximum principle for weakly L-subharmonic functions
Professor Walter Littman, University of California, Berkeley (553-119)

(38) On absolutely convergent exponential sums
Professor Leon Brown, Wayne State University, Professor A. L. Shields, University of Michigan, and Dr. Karl Zeller, University of Tübingen (553-80)

(39) On the L^1 norm of exponential sums
Professor P. J. Cohen, Massachusetts Institute of Technology (553-8)

(40) Multiplicative summability methods and the Stone-Čech compactification
Professor Melvin Henriksen, Purdue University (553-86)

(41) On functional calculus
Dr. Chandler Davis, American Mathematical Society, Providence, Rhode Island (553-137)

Session on Geometry, Physical Sciences Building, Auditorium A-2

(42) Regular transformations in Euclidean 4-space
Professor Simon Green and Mr. W. A. Rutledge, University of South Carolina (553-30)

(43) Close-packing and froth
Professor H. S. M. Coxeter, University of Toronto (553-83)

(44) On planes of order nine
Professor J. D. Swift, University of California, Los Angeles, Professor Marshall Hall, Jr., Ohio State University, and Mr. R. Killgrove, University of California, Los Angeles (553-56)

(45) Polynomials associated with divisors
Professor Ernst Snapper, Indiana University (553-55)

(46) Some properties of certain pencils of conics
Professor S. R. Mandan, Indian Institute of Technology, and Professor Seymour Schuster, Carleton College and Polytechnic Institute of Brooklyn (553-74)

(47) On the projective centers of convex curves
Professor Paul Kelly, University of California, Santa Barbara, and Professor E. G. Straus, University of California, Los Angeles (553-16)

Session on Algebra and Theory of Numbers, Physical Sciences Building Auditorium A-4

(48) The algebra of a linear hypothesis
Professor H. B. Mann, Ohio State University (553-50)

(49) On characteristically nilpotent Lie algebras
Professor G. F. Leger, University of Pittsburgh, and Mr. Shigeaki Tôgô, Northwestern University and Hiroshima University (553-118)

(50) Nodal noncommutative Jordan algebras and simple Lie algebras of characteristic p
Professor R. D. Schafer, University of Connecticut and Institute for Advanced Study (553-97)

(51) Algebras with weak dimension equal zero
Mr. O. E. Villamayor, Northwestern University and Universidad Nacional de La Plata
(Introduced by Professor R. D. Schafer)

(52) Nilalgebras and linear varieties of nilpotent matrices. IV
Professor Murray Gerstenhaber, Institute for Advanced Study and University of Pennsylvania (553-46)

(53) Basic algebras of UMFR algebras
Professor D. W. Wall, University of North Carolina (553-129)

(54) Linear transformations on algebras of matrices. I
Professor M. D. Marcus and Professor B. N. Moyls, University of British Columbia (553-44)

(55) Differentiability and analyticity of functions in linear algebras
Dr. Anthony Trampus, General Electric Company, Cincinnati, Ohio (553-36)

Session on Applied Mathematics, Moore School Building, Auditorium

(56) Best group alphabets
Dr. Isidore Fleischer, Brooklyn, New York (553-21)

(57) Application of modular sequential circuits to single error-correcting p-nary codes
Professor T. E. Stern and Professor Bernard Friedland, Columbia University (553-57)
(Introduced by Professor L. A. Zadeh)

(58) Noncooperative games and nonlinear programs
Dr. H. D. Mills, Market Research Corporation of America and Princeton University (553-51)

(59) A decomposition principle for linear programs
Dr. G. B. Dantzig, RAND Corporation, Santa Monica, California (553-59)

(60) Integral geometric methods in information theory. I. Invariant measures
Professor W. R. Baum, Syracuse University (553-76)

(61) On the inversion of certain matrices
Dr. Samuel Schechter, New York University (553-10)

(62) Algorithm for Chebycheff approximations using the ratio of linear forms. Preliminary report
Mr. H. L. Loeb, CONVAIR Astronautics, San Diego, California (553-120)
(Introduced by Dr. H. E. Salzer)

TUESDAY, 8:00 P.M.

Gibbs Lecture, University Museum, Auditorium

On the emergence of patterns of order (One hour)
Professor J. M. Burgers, University of Maryland

WEDNESDAY, 10:00 A.M.

Session on Analysis, Physical Sciences Building, Auditorium A-1

(63) On the existence of certain Green's functions
Dr. J. W. Neuberger, Illinois Institute of Technology (553-66)

(64) Some new external problems with infinitely many interpolating conditions
Dr. D. S. Greenstein, University of Michigan (553-7)

(65) A theorem on integral transforms with an application to prediction theory
Professor Joshua Chover, University of Wisconsin (553-27)

(66) A problem of least area
Professor Edward Silverman, Purdue University (553-151)

(67) On upper and lower limits in measure
Professor Casper Goffman and Professor Daniel Waterman, Purdue University (553-61)

(68) On functions which satisfy rational recurrence formulae and related classes of real numbers. Preliminary report
Professor H. S. Shapiro, New York University (553-99)

(69) Invariant positive linear functionals
Professor Joseph Kist, Wayne State University (553-34)

(70) On the asymptotic behaviour of a differential-functional equation
Dr. R. B. Kelman, IBM Research Center, Yorktown, New York (553-117)

(71) The problem of extension of quasi additive set functions
Professor Lamberto Cesari, Purdue University

Session on Analysis, Physical Sciences Building, Auditorium A-2

(72) On the maximality of vanishing algebras
Mr. A. B. Simon, Yale University (553-41)

(73) Continuity properties of homomorphisms of function algebras. Preliminary report
Professor W. G. Bade, University of California, Berkeley and Yale University, and Professor P. C. Curtis, Jr., University of California, Los Angeles and Yale University (553-109)

(74) Probabilities on a compact group
Dr. Karl Stromberg, Yale University (553-14)

(75) Some theorems on lacunary Fourier series, with extensions to compact groups
Professor Edwin Hewitt and Professor H. S. Zuckerman, University of Washington (553-87)

(76) Functions which operate on Fourier-Stieltjes transforms
Professor Walter Rudin, University of Rochester and Yale University (553-149)

(77) Almost periodic motions in uniform spaces
Dr. G. H. Meisters, Duke University (553-35)

(78) Differentiation on Riemann surfaces
Professor W. C. Fox, Northwestern University (553-113)

(79) On Riemann's problem of function theory
Professor J. C. C. Nitsche, University of Minnesota (553-124)

(80) Meromorphic modifications
Dr. W. F. Stoll, Institute for Advanced Study (553-128)

Session on Algebra and Theory of Numbers, Physical Sciences Building, Auditorium A-4

(81) Construction of some sets of pairwise orthogonal latin squares. Preliminary report
Dr. E. T. Parker, Remington Rand UNIVAC, St. Paul, Minnesota (553-67)

(82) An abstract definition for the Mathieu group of degree twelve
Mr. William Moser, University of Saskatchewan (553-20)

(83) On certain totally ordered non-Abelian groups of rank 2
Professor N. L. Alling, Purdue University (553-39)

(84) On groups where cosets are classes
Professor H. W. E. Schwerdtfeger, McGill University (553-98)

(85) The reduction of an ordinary representation of S_n into its modular components for $p \leqq n$
Miss Diane Johnson and Professor G. de B. Robinson, University of Toronto (553-104)

(86) Some generalized Frattini subgroups of finite groups. Preliminary report
Professor W. E. Deskins, Michigan State University (553-111)

(87) The representation of finite groups in algebraic number fields
Mr. Louis Solomon, Bryn Mawr College (553-127)

(88) Some theorems on Moufang loops
Professor R. H. Bruck, University of Wisconsin (553-70)

(89) A new class of neofields
Dr. J. M. Osborn, University of Wisconsin (553-148)

Session on Applied Mathematics, Moore School Building, Auditorium

(90) A maximum theorem for irrotational subsonic gas flow
Dr. Barry Bernstein, Naval Research Laboratory, Washington, D. C. (553-15)

(91) Acoustic streaming past a sphere in a viscous, compressible and heat conducting fluid. Preliminary report
Dr. R. P. Kanwal, University of Wisconsin (553-33)

(92) One-dimensional heat flow with space-varying thermal properties
Mr. M. S. Klamkin, Avco Research and Advanced Development Division, Wilmington, Massachusetts (553-65)

(93) On the Lorenz potentials
Professor Domina E. Spencer, University of Connecticut (553-102)

(94) Appell's equations in relativistic particle dynamics
Dr. L. A. MacColl, Bell Telephone Laboratories, New York, New York (553-17)

(95) A model of turbulence displaying autonomous oscillations. Preliminary report
Dr. R. J. Dickson, Lockheed Missile Systems Division, Palo Alto, California (553-84)

(96) On the theory of a class of non-self-adjoint operators and its applications to quantum scattering theory
Professor C. L. Dolph, University of Michigan, and Dr. F. Penzlin, University of Munich (553-2)

(97) On a new class of integral transforms
Dr. Ta Li, CONVAIR Astronautics, San Diego, California and University of California, Los Angeles (553-5)

WEDNESDAY, 2:00 P.M.

Invited Address, University Museum, Auditorium
The spectral theory of partial differential operators (One hour)
Professor Felix Browder, Yale University

WEDNESDAY, 3:15 P.M.

Session on Analysis, Physical Sciences Building, Auditorium A-1

(98) Dual variational problems
Dr. H. B. Jenkins, New York University (553-88)

(99) General boundary value problems for elliptic partial differential equations. III
Dr. Martin Schechter, New York University (553-1)

(100) Boundary value problems for hyperbolic equations
Professor P. C. Rosenbloom, University of Minnesota (553-114)

(101) Free boundary problems for parabolic equations
Professor Avner Friedman, University of California, Berkeley (553-12)
(Introduced by Professor C. B. Morrey)

(102) On a nonlinear parabolic equation
Professor W. B. Fulks, University of Minnesota, and Professor J. S. Maybee, New York University and University of Southern California (553-85)

(103) On estimation and uniqueness of solutions of a mixed problem for non-linear, parabolic, second order partial differential equations
Professor Jacek Szarski, University of Cracow and University of Kansas
(Introduced by Professor Nachman Aronszajn)

(104) A class of non-self-adjoint boundary conditions for the Laplacian. Preliminary report
Professor W. G. Bade, University of California, Berkeley (553-134)

(105) Singular second order differential boundary problem of Fourier type
Professor R. W. McKelvey, University of Colorado (553-121)

Session on Topology, Physical Sciences Building, Auditorium A-2

(106) On the cohomology of a space on which an H-space operates
Dr. Edward Halpern, University of Michigan (553-9)

(107) Suspensions in loop spaces
Dr. William Browder, Cornell University (553-79)

(108) Universal coefficient system for cohomology rings
Dr. F. P. Palermo, University of Michigan (553-94)

(109) On generalized Vietoris complexes and nerve complexes. Preliminary report
Mr. J. E. Keisler, University of Michigan (553-89)

(110) Homogeneous fibre structures
Professor Andrew Sobczyk, University of Florida (553-152)

(111) On limits of module-systems
Professor Johann Sonner, University of South Carolina (553-38)

(112) Ideals in rings of continuous functions
Professor T. R. Jenkins, Washington State College, and Dr. J. D. McKnight, Jr., Lockheed Missile Systems Division, Palo Alto, California (553-116)

Session on Algebra and Theory of Numbers, Physical Sciences Building, Auditorium A-4

(113) On regular bilinear forms
Professor Peter Scherk, University of Saskatchewan (553-54)

(114) Quadratic forms over rings
Professor Barth Pollak, Syracuse University (553-126)

(115) Integral bases in Kummer extensions of algebraic number fields. Preliminary report
Mr. L. R. McCulloh, Ohio State University (553-49)

(116) Diophantine equations in quadratic number-fields
Miss Paromita Chowla, University of Colorado (553-75)
(Introduced by Professor Sarvadaman Chowla)

(117) Means of arithmetic functions
Professor S. A. Amitsur, Hebrew University and University of Notre Dame (553-105)

(118) On the minimal overlap problem of P. Erdös
Professor Leo Moser, University of Alberta (553-123)

(119) Polynomial identities. I
Professor J. B. Roberts, Reed College (553-11)

Session on Logic and Foundations, Moore School Building, Auditorium

(120) On the formalization of the concept of internal relation
Dr. F. G. Asenjo, Georgetown University (553-107)
(Introduced by Professor M. W. Oliphant)

(121) Decision problems of weak second order arithmetics and finite automata. Preliminary report. Part I
Mr. C. C. Elgot and Professor J. R. Büchi, University of Michigan (553-112)

(122) Some consequences of the axiom of constructibility
Professor J. W. Addison, University of Michigan (553-138)

(123) Generalized continuity
Professor P. C. Hammer, University of Wisconsin (553-141)

(124) Direct products of algebras
Professor R. C. Lyndon, University of Michigan (553-145)

(125) Strongly invariant hierarchies. Preliminary report
Professor Clifford Spector, Ohio State University (553-153)

Session on Analysis, Physical Sciences Building, Auditorium A-1

(126) Maximal vector spaces of light, interior, orientation-preserving, C' functions
Mr. W. V. Caldwell, University of Michigan (553-82)

(127) A reflection theorem for light interior functions
Professor G. S. Young, University of Michigan (553-130)

(128) A note on entire functions. Preliminary report
Dr. S. K. Singh, University of Kansas (553-26)
(Introduced by Professor George Springer)

(129) Meromorphic functions with maximum defect
Professor S. M. Shah, University of Wisconsin, and Dr. S. K. Singh, University of Kansas (553-150)

(130) Nearest functions
Dr. Oved Shisha, Harvard University and Technion-Israel Institute of Technology (553-158)
(Introduced by Professor J. L. Walsh)

(131) A non-Euclidean analogue to a theorem of H. Milloux and its relationship to a theorem of W. Seidel
Professor L. H. Lange, University of Notre Dame (553-144)

(132) Extremal problems for starlike functions
Professor J. A. Hummel, University of Maryland (553-63)

(133) Boundary behavior of Blaschke products
Mr. G. T. Cargo, University of Michigan (553-42)

(134) On certain automorphic functions
Mr. M. I. Knopp, Space Technology Laboratories, Inc., Los Angeles, California (553-32)

Session on Analysis, Physical Sciences Building, Auditorium A-2

(135) Intersections of cells in normed spaces
Dr. Branko Grünbaum, Institute for Advanced Study (553-73)
(Introduced by Professor Marston Morse)

(136) Absolutely continuous homeomorphism groups in Hilbert space
Dr. Leonard Gross, Yale University

(137) On compact linear transformations in Banach space
Professor C. T. Taam, Georgetown University (553-154)

(138) A spectral resolution for a bounded linear operator some power of which is normal
Mr. J. G. Stampfli, University of Michigan (553-103)

(139) A space of multipliers of Fourier integrals
Professor G. L. Krabbe, Purdue University (553-48)

(140) Projections of separable spaces of bounded sequences onto the space of convergent sequences
Dr. R. D. McWilliams, Princeton University (553-122)

(141) Limit-convergence of function sequences
Professor Guy Johnson, Rice Institute (553-142)

(142) Almost uniform convergence versus pointwise convergence
Professor J. W. Brace, University of Maryland (553-78)

(143) Weak convergence and weak compactness in the space of functions of bounded variation
Professor Pasquale Porcelli, University of Wisconsin (553-68)

Session on Geometry, Physical Sciences Building, Auditorium A-4

(144) On a problem of Blaschke
Professor L. W. Green, University of Minnesota (553-43)

(145) Wave equations on homogeneous spaces
Dr. Sigurdur Helgason, University of Chicago (553-115)

(146) On differentiability properties of Kahler metrics
Dr. A. W. Adler, Massachusetts Institute of Technology, and Mr. J. J. Kohn, Brandeis University (553-6)

(147) Hyperconvexity in Riemannian manifolds
Professor Albert Nijenhuis, University of Washington (553-146)

(148) Projective-geodesic mapping of surfaces. Preliminary report
Dr. Paolo Lanzano, Space Technology Laboratories, Los Angeles, California (553-90)

(149) Sheets of real analytic varieties
Professor A. H. Wallace, University of Toronto (553-31)

(150) The differential invariants of local regular surface mappings
Professor Gabriel Margulies, Florida State University (553-160)

Session on Statistics and Probability, Moore School Building, Auditorium

(151) Pointwise convergence of martingales with a directed index set
Dr. Y. S. Chow, University of Illinois (553-45)

(152) Some attraction properties of the Poisson distribution
Professor J. M. Shapiro, Ohio State University (553-100)

(153) On the number of changes in sign
Professor G. E. Baxter, University of Minnesota (553-110)

(154) Statistical inference on time series by Hilbert space methods. I
Professor Emanuel Parzen, Stanford University (553-125)

(155) Subordination of infinitely divisible processes
Professor J. W. Woll, Jr., University of California, Berkeley (553-157)

(156) On sequences of events
Professor Louis Sucheston, University of Rochester (553-159)

THURSDAY, 2:00 P.M.

Bôcher Prize; Annual Business Meeting, University Museum, Auditorium

THURSDAY, 3:15 P.M.

Session on Analysis, Physical Sciences Building, Auditorium A-1

(157) L-regular matrices
Professor G. M. Petersen, University of New Mexico (553-95)

(158) Total comparison between two Cesàro matrices
Professor B. E. Rhoades, Lafayette College (553-96)

(159) Semi-groups of Toeplitz matrices
Professor W. B. Jurkat, Syracuse University (553-143)

(160) Rademacher series with geometric coefficients. Preliminary report
Mr. W. A. Beyer, Pennsylvania State University (553-131)

(161) The Lebesgue constants for Jacobi series
Professor Lee Lorch, Philander Smith College and Wesleyan University (553-71)

(162) On the Euler and Taylor summation of Dirichlet and Taylor series
Professor V. F. Cowling and Professor W. C. Royster, University of Kentucky (553-136)

Session on Applied Mathematics, Moore School Building, Auditorium

(163) Power series for the n-body problem
Dr. Ward Cheney, CONVAIR Astronautics, San Diego, California (553-18)

(164) Taylor series for the regularized N-body problem
Dr. A. A. Goldstein, CONVAIR Astronautics, San Diego, California (553-37)

(165) Expansions of parabolic wave functions
Professor Yousef Alavi, Western Michigan University, and Professor C. P. Wells, Michigan State University (553-58)

(166) Spherical means and radiation conditions
Professor C. H. Wilcox, University of Wisconsin (553-69)

(167) A nonlinear delay-differential equation
Professor J. A. Nohel, University of Wisconsin and Georgia Institute of Technology (553-93)

(168) Hoelder continuity and initial value problem of mixed type differential equations
Professor Y. W. Chen, Wayne State University (553-81)

Sessions for Late Papers

SUPPLEMENTARY PROGRAM
(To be presented by title)

(169) Sums of normal endomorphisms
Professor R. H. Bruck, University of Wisconsin

(170) An open question concerning Moufang loops
Professor R. H. Bruck, University of Wisconsin

(171) Characterizations of tree-like continua
Professor J. H. Case and Professor R. E. Chamberlin, University of Utah

(172) Proximity maps for convex sets
Dr. Ward Cheney and Dr. A. A. Goldstein, CONVAIR Astronautics, San Diego, California

(173) A duality theory of locally compact maximally almost periodic groups
Mr. Hsin Chu, University of Pennsylvania

(174) Idempotent measures on locally compact abelian groups
Dr. P. J. Cohen, Massachusetts Institute of Technology

(175) On homomorphisms of group algebras
Dr. P. J. Cohen, Massachusetts Institute of Technology

(176) The tensor product of polyadic algebras
Mr. Aubert Daigneault, Royal Military College of Canada

(177) Webster's horn theory
Dr. R. J. Dickson, Lockheed Missile Systems Division, Palo Alto, California

(178) Rotational nonviscous flows in two dimensions
Dr. R. J. Dickson, Lockheed Missile Systems Division, Palo Alto, California

(179) A variant of Abel's lemma
Dr. R. J. Dickson, Lockheed Missile Systems Division, Palo Alto, California

(180) Variational principles for point source scattering problems
Professor C. L. Dolph, University of Michigan

(181) A note on piercing a disk
Professor P. H. Doyle and Professor J. G. Hocking, Michigan State University

(182) Extended pth-powers in H-spaces
Professor Eldon Dyer and Professor R. K. Lashof, University of Chicago

(183) Some remarks on commutative algebras of operators on Banach spaces
Dr. D. A. Edwards and Mr. C. I. Tulcea, Yale University

(184) On equationally definable classes of algebras
Mr. C. C. Elgot, University of Michigan

(185) Decision problems of weak second order arithmetics and finite automata. Preliminary report. Part II
Mr. C. C. Elgot, University of Michigan

(186) Equicontinuity and almost periodic functions
Professor Robert Ellis, University of Pennsylvania

(187) A semigroup associated with a transformation group
Professor Robert Ellis, University of Pennsylvania

(188) Homomorphisms of transformation groups
Professor Robert Ellis and Professor W. H. Gottschalk, University of Pennsylvania

(189) On the uniqueness of the Cauchy problem for parabolic equations
Professor Avner Friedman, University of California, Berkeley
(Introduced by Professor C. B. Morrey)

(190) Generalized heat transfer between solids and gases under nonlinear boundary conditions
Professor Avner Friedman, University of California, Berkeley
(Introduced by Professor C. B. Morrey)

(191) Characteristics of Monge differential equations as the curves of least effect in the spatial field of force (with a philosophical introduction)
Professor O. E. Glenn, Lansdowne, Pennsylvania

(192) Solution of the integral equation for the solar surface brightness
Dr. A. A. Goldstein, CONVAIR Astronautics, San Diego, California

(193) On best nine-point difference approximations of Laplace's equation
Professor Donald Greenspan, Purdue University

(194) On the numerical solution of boundary value problems of mixed type
Professor Donald Greenspan, Purdue University

(195) On the primitivity of Hopf algebras over a field with prime characteristic
Dr. Edward Halpern, University of Michigan

(196) On the line-group of two-terminal series-parallel networks
Professor Frank Harary, University of Michigan, Princeton University, and Institute for Advanced Study

(197) On the group of a graph with respect to a subgraph
Professor Frank Harary, University of Michigan, Princeton University, and Institute for Advanced Study

(198) A theorem on general branching processes. Preliminary report
Dr. T. E. Harris, RAND Corporation, Santa Monica, California

(199) Two-point homogeneous spaces
Dr. Sigurdur Helgason, University of Chicago

(200) Determination of a function from its integrals over totally geodesic sub-manifolds
Dr. Sigurdur Helgason, University of Chicago

(201) "Development" of curves. Preliminary report
Dr. Robert Hermann, Harvard University

(202) "Natural" lifting of curves. Preliminary report
Dr. Robert Hermann, Harvard University

(203) A structure theory for a class of lattice-ordered rings. II Preliminary report
Mr. D. G. Johnson, Purdue University

(204) Full Banach mean values on countable groups. Preliminary report
Dr. Harry Kesten, Princeton University

(205) Quantification of number-theoretic functions
Professor S. C. Kleene, University of Wisconsin

(206) Spectral representations for finite Hilbert transformations
Dr. Walter Koppelman, Yale University, and Mr. J. D. Pincus, New York University

(207) The imbeddability of the real projective spaces in Euclidean space
Professor W. S. Massey, Brown University

(208) Power series representing rational functions
Professor Z. A. Melzak, McGill University

(209) On Lagrange stable motions in the neighborhood of critical points
Dr. Pinchas Mendelson, Columbia University

(210) On dynamical systems without improper saddle points. I
Dr. Pinchas Mendelson, Columbia University

(211) Typically-real functions in a cut plane
Professor E. P. Merkes, De Paul University

(212) Two-person noncooperative game solutions
Dr. H. D. Mills, Market Research Corporation of America and Princeton University

(213) Recurrent Markov chains. I
Professor Steven Orey, University of Minnesota

(214) On the solution of a certain non-linear differential equation of the second order
Professor T. J. Pignani and Mr. D. I. Koehler, University of Kentucky

(215) Slight generalizations of a theorem of Levitzki
Dr. E. C. Posner, University of Wisconsin

(216) The number of periodic solutions of non-autonomous systems
Professor Jane Cronin Scanlon, Polytechnic Institute of Brooklyn

(217) Some connections between continued fractions and convex sets
Mr. Robert Seall, Ohio State University, and Professor Marion D. Wetzel, Denison University

(218) Meromorphic functions of integer order
Professor S. M. Shah, University of Wisconsin

(219) On homotopy associativity of H-spaces. Preliminary report
Mr. J. D. Stasheff, Princeton University

(220) On certain binary reaction systems
Mr. P. R. Stein and Dr. S. M. Ulam, University of California, Los Alamos

(221) Regular automorphisms
Professor S. K. Stein, University of California, Davis

(222) A weak property L for pairs of matrices
Dr. Olga Taussky, California Institute of Technology

(223) On the analogue for maximally convergent polynomials of Jentzsch's theorem
Professor J. L. Walsh, Harvard University

(224) The problem of N traveling salesmen
Dr. L. E. Ward, Jr., Naval Ordnance Test Station, China Lake, California

(225) On the eigenvalues of certain translation kernels
Professor Harold Widom, Cornell University

(226) A surface having no curves of finite length
Professor R. F. Williams, Purdue University

R. D. Schafer
Associate Secretary

Princeton, New Jersey
December 5, 1958

PRELIMINARY ANNOUNCEMENT OF MEETING

FIVE HUNDRED FIFTY-FOURTH MEETING

Columbia University
New York, New York
February 28, 1959

The five hundred fifty-fourth meeting of the American Mathematical Society will be held at Columbia University in New York City on Saturday, February 28, 1959. All sessions will be held in the Pupin Physical Laboratories.

Professor John Nash of the Massachusetts Institute of Technology will deliver an address entitled "An unorthodox view of the Riemann hypothesis" in Room 301 at 2:00 P.M. by invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings.

Sessions for contributed papers will be held at 10:00 A.M. and at 3:15 P.M. A registration table will be found near the place of the meeting.

Columbia University may be reached by the Broadway-7th Avenue line of the IRT Subway at the 116th Street station. The Pupin Physical Laboratories are on the south side of 120th Street at Broadway.

Further details of the meeting will appear in the next issue of the NOTICES. Abstracts of contributed papers should be sent to the American Mathematical Society, 190 Hope Street, Providence 6, Rhode Island, so as to arrive PRIOR TO THE DEADLINE, January 15. It is expected that abstracts of all papers to be presented in person will appear in the same issue of the NOTICES, but only abstracts which meet the specifications stated on the abstract blanks can be published.

R. D. Schafer
Associate Secretary

Princeton, New Jersey
December 5, 1958

ACTIVITIES OF OTHER ASSOCIATIONS

THE FORTY-SECOND ANNUAL MEETING OF THE MATHEMATICAL ASSOCIATION OF AMERICA will be held at the University of Pennsylvania, Philadelphia, Pennsylvania, on Thursday and Friday, January 22 and 23, 1959, in conjunction with meetings of the American Mathematical Society, The Association for Symbolic Logic, and the Delaware Valley Section of the Society for Industrial and Applied Mathematics. Sessions of the Mathematical Association will be held on Thursday at 9:00 A.M. and on Friday at 9:00 P.M. and 2:00 P.M. in the Auditorium of the University Museum, at 34th and Spruce Streets, entrance through 34th Street. At the annual business meeting of the Association on Friday afternoon, motions will be voted upon to amend the By-Laws of the Association as described on page 727 of the November 1958 issue of the American Mathematical Monthly. The Board of Governors of the Association will meet on Thursday afternoon at 3:30 in the Poor Richard Room of the Benjamin Franklin Hotel.

THURSDAY: 9:00 A.M.

First Session, Auditorium, University Museum
"Professional Opportunities in Mathematics"
Moderator: Dr. M. A. Shader, IBM Corporation
Panel Members: Professor Wallace Givens, Wayne State University
Dr. H. H. Goldstine, IBM Corporation
Dr. J. P. Nash, Lockheed Aircraft Corporation

FRIDAY: 9:00 A.M.

Second Session, Auditorium, University Museum
"The Training of Secondary School Mathematics Teachers"
Moderator: Professor J. G. Kemeny, Dartmouth College
Panel Members: Professor C. B. Allendoerfer, University of Washington
Professor H. F. Fehr, Teachers College, Columbia University
Professor E. P. Northrop, University of Chicago
Professor H. E. Vaughan, University of Illinois

FRIDAY: 2:00 P.M.

Third Session, Auditorium, University Museum
Annual Business Meeting of the Association
"High School Mathematics Courses"
(Co-sponsored by the School Mathematics Study Group)

Moderator: Professor E. G. Begle, Yale University and School Mathematics Study Group

Panel Members: Dr. J. R. Mayor, American Association for the Advancement of Science
Professor E. E. Moise, University of Michigan
Dr. H. O. Pollak, Bell Telephone Laboratories
Professor G. B. Price, University of Kansas
Mr. R. E. K. Rourke, Kent School and Commission on Mathematics
Professor D. E. Richmond, Williams College

THE DELAWARE VALLEY SECTION OF THE SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS will present a talk by Dr. John Mauchly of Remington-Rand Univac on Wednesday, January 21, 1959, at 8:00 P.M. in the Physical Sciences Building of the University of Pennsylvania. The title of Dr. Mauchly's talk is "The Role of the Mathematician in Industry". The lecture will be followed by an informal social get-together to be held in the Rotunda of the University of Pennsylvania Museum. Anyone is welcome at both the talk and the get-together. There will be a charge of $1.00 for the party and beer, soda, pretzels and potato chips will be served. Tickets for this party will be available at the registration desk.

NEWS ITEMS AND ANNOUNCEMENTS

THE C. L. E. MOORE INSTRUCTORSHIPS AT M. I. T. The Department of Mathematics at the Massachusetts Institute of Technology wishes to announce the availability of C. L. E. Moore Instructorships in Mathematics for 1959-1960, open to young mathematicians with doctorates who show definite promise in research. The base salary for these instructorships is $6,500 and the teaching load will be six hours per week. The salary can be supplemented by summer work on research contracts or by teaching in the summer session. The appointments are annual but may be renewed twice.

Applications should be filed not later than January 26, 1959 on forms obtained from the Department.

POSTDOCTORAL RESIDENT RESEARCH ASSOCIATESHIPS of the NAS-NRC. The National Academy of Sciences - National Research Council has announced a program of Postdoctoral Resident Research Associateships to be offered for 1959-1960. The participating laboratories are the National Bureau of Standards (Boulder, Colorado and Washington, D. C.); the Naval Ordnance Laboratory (White Oak, Silver Spring, Maryland); the Naval Research Laboratory (Washington, D. C.); the Navy Electronics Laboratory (San Diego, California); and the U. S. Army Chemical Corps Biological Warfare Laboratories (Fort Detrick, Frederick, Maryland).

The Air Research and Development Command is also participating in this program at four Air Force installations. These associateships are tenable at Air Force Cambridge Research Center (Bedford, Massachusetts); Air Force Missile Development Center (Alamogordo, New Mexico); Rome Air Development Center (Rome, New York); and Wright Air Development Center (Dayton, Ohio). In addition, the ARDC is sponsoring a program of Postdoctoral University Research Associateships tenable at twenty-one universities in the United States.

The resident research associateships have been established to provide young scientists of unusual ability and promise an opportunity for advanced training in basic research in a variety of fields. Modern facilities are available in specified areas of the biological, physical, and mathematical sciences, and engineering. In addition to the above research in certain areas of psychology is available.

Applicants must be citizens of the United States. They also must produce evidence of training in one of the listed fields equivalent to that represented by the Ph. D. or Sc. D. degree and must have demonstrated superior ability for creative research. Remuneration for these associateships is from $5985 to $7510 a year subject to income tax.

Application materials may be secured by writing to the Fellowship Office, National Academy of Sciences - National Research Council, 2101 Constitution Avenue, N. W., Washington 25, D. C. In order to be considered for awards for 1959-1960, applications must be filed at the Fellowship Office on or before January 19, 1959. Awards will be announced about April 1, 1959 by the participating laboratories and research centers.

BORIS A. BAKHMETEFF RESEARCH FELLOWSHIP IN MECHANICS OF FLUIDS OFFERED BY HUMANITIES FUND, INC., NEW YORK, NEW YORK. A Boris A. Bakhmeteff Research Fellowship will be available for the 1959-1960 academic year in an amount up to $3000. It is intended to be a specific contribution for a definite research project of an original and creative nature in the general field of mechanics of fluids.

The recipient shall be a full-time graduate student who is a candidate for the master's or doctoral degree. He shall not hold, or expect to hold, any other fellowship or income-producing commitment that will interfere with his research work and study on a full-time basis. It is expected that the stipend will be adequate to cover tuition, subsistence, and, if necessary, a portion of the research expenses.

The study and research may be undertaken at an institution of the Fellow's choice. In the judgment of the Committee, the adequacy of the facilities of the institution will have substantial weight in the selection of the Fellow.

Applications should be filed by February 15, 1959 with Dean William Allan, School of Technology, The City College of New York, New York 31, New York.

Committee of Consultants is as follows: William Allan, The City College of New York; Morrough P. O'Brien, University of California; and Lorenz G. Straub, University of Minnesota.

PREDOCTORAL AND POSTDOCTORAL FELLOWSHIPS IN THE SCIENCES AND ENGINEERING OF THE NATIONAL SCIENCE FOUNDATION. The National Science Foundation announced today that applications are being accepted in two National Science Foundation fellowship programs for advanced study in the natural sciences. The two fellowship programs are:

(1) a predoctoral fellowship program for which college seniors and graduate students may apply; and

(2) a postdoctoral fellowship program for scientists who have already received the doctoral degree.

Approximately 1100 awards will be made in these two programs in March 1959. These awards are in addition to 34 Postdoctoral awards offered in October 1958, and approximately 75 Senior Postdoctoral and 300 Science Faculty fellowships to be awarded in December 1958.

National Science Foundation fellowships are awarded to citizens of the United States who have demonstrated special aptitude in science and who will begin or continue their studies at the graduate or postdoctoral level. Under the broadened program fellowships will be offered in the mathematical, physical, medical, biological, and engineering sciences, as well as in anthropology, psychology (other than clinical), and in certain selected social sciences. The social sciences included are geography, mathematical economics, econometrics, demography, information and communication theory, experimental and quantitative sociology and the history and philosophy of science where they conform to accepted standards of scientific inquiry by fulfilling the requirements of the basic scientific method as to objectivity, verifiability, and generality.

Fellows will be selected on the basis of ability as evidenced by letters of recommendation, academic records, and other evidences of attainment. Applicants for the predoctoral fellowships are required to take the Graduate Record Examinations. Candidates' qualifications will be evaluated by carefully chosen panels selected by the National Academy of Sciences-National Research Council. Final selection of Fellows will be made by the National Science Foundation.

Stipends for National Science Foundation fellowships vary with the academic status of the Fellows. First-year Fellows - students entering graduate school for the first time or those who have had less than one year of graduate study - will receive annual stipends of $1800. Fellows who need one final academic year of training for the doctoral degree will receive annual stipends of $2200. Fellows between these two groups will receive stipends at the rate of $2000 annually. The stipends for the regular Postdoctoral Fellows will be $4500 per year. Dependency allowance of $500 per annum for spouse and for each dependent child will normally be available. Tuition and laboratory fees and limited travel allowances will also be provided.

National Science Foundation Fellows may attend any accredited nonprofit institutions of higher education in the United States or nonprofit institutions of higher education abroad.

Applications for the National Science Foundation graduate and postdoctoral fellowship programs may be obtained from the Fellowship Office, National Academy of Sciences-National Research Council, 2101 Constitution Avenue, N. W., Washington 25, D. C. The closing dates for the receipt of applications are December 22, 1958, for postdoctoral applicants and January 5, 1959, for graduate students working toward advanced degrees in science. The selections will be announced March 15, 1959.

Editor's Note: The deadline for tne November NOTICES was October 8. In the latter part of October and in November, several announcements of National Science Foundation fellowship opportunities, such as the above announcement, were received in the editorial office of

the NOTICES carrying December closing dates for receipt of applications. Since this present issue of the NOTICES is not expected to reach the subscribers before the end of December, full coverage is not being given here to these announcements. More generally, we regret to say that as long as the National Science Foundation sees fit to announce many of its fellowships only four to eight weeks in advance of closing dates for application, it will never be possible to give adequate coverage to these announcements in the NOTICES.

EDUCATIONAL TESTING SERVICE VISITING ASSOCIATESHIPS IN TEST DEVELOPMENT FOR SCHOOL AND COLLEGE TEACHERS FOR THE SUMMER OF 1959. Educational Testing Service will offer two Visiting Associateships in Test Development for the Summer of 1959, one in Mathematics and one in Science. The Associateships will give experienced teachers an opportunity to study testing problems in relation to goals of instruction. In their work with the Test Development staff, the Associates will become familiar with testing techniques, and at the same time they will bring frontline experience to bear on the problems of evaluation in nation-wide testing programs. The duration of the program is from June 30 to August 28, 1959. The stipend is $700, with reimbursement for transportation to and from Princeton. Both Associates will make critical analyses of existing test specifications and test questions, suggest improvements, and work on the preparation of new tests. They will work on tests at the college-entrance and higher levels. The qualifications are as follows: The Visiting Associate in Science should have a strong background in chemistry, physics, or biology. Training in more than one of these sciences is desirable. The Associate should have four or more years of teaching experience in college or in secondary school, or in the two combined. The Visiting Associate in Mathematics should have a strong background in modern mathematics. Four or more years of college teaching experience is required. In selecting the Associates, emphasis will be placed on academic scholarship and on successful teaching experience. Individuals who are interested in the broad problems of educational measurement will receive preference, although specialized training in this area is not expected.

Applications, to be submitted by February 27, 1959, should include a completed application form and transcripts of all college work, both graduate and undergraduate. Requests for application forms, completed applications, and all inquiries should be addressed to Mrs. W. Stanley Brown, Test Development Division, Educational Testing Service, 20 Nassau Street, Princeton, New Jersey. Applications should be submitted by February 27, 1959. Appointments will be made by March 23, 1959.

THE INSTITUTE FOR FLUID DYNAMICS AND APPLIED MATHEMATICS OF THE UNIVERSITY OF MARYLAND expects to make a limited number of Visiting Research appointments for the academic year 1959-60. Further information may be obtained by writing to the Director, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland.

THE UNIVERSITY OF ARIZONA announces a new Ph. D. program in mathematics. A limited number of assistantships are available. Further information can be obtained by writing to Dr. Harvey Cohn, Head, Department of Mathematics, University of Arizona, Tucson, Arizona.

SUMMER TRAINING PROGRAMS FOR SECONDARY SCHOOL STUDENTS OF THE NATIONAL SCIENCE FOUNDATION. As part of its experimental program in encouraging the scientific interests of high-ability secondary school students, the National Science Foundation will support a limited number of proposals from colleges, universities and other non-profit research and higher educational institutions to provide, during the summer of 1959, opportunities for such students to study and work with experienced scientists and mathematicians. No fixed pattern is prescribed for such programs. Each institution is encouraged to develop its own method of achieving the desired results in accordance with available facilities and personnel. It is anticipated that a variety of programs will be supported, including but not limited to one or more of the following: classwork, field trips, orientation lectures, laboratory visits and research participation activities. Proposals sponsored by secondary schools are not eligible under this program. However, the facilities of secondary schools can be utilized in college-sponsored off-campus programs where this is mutually acceptable.

Under this program Foundation support may include the expenses of some or all of the student participants for room, board, travel (including commuting) and other essential items when appropriate; direct costs of the sponsoring institutions such as fractions of staff salaries properly attributable to the program, payments to high school science teachers - or individuals who are studying to become science teachers - used as counselor-participants, and other necessary expenses and supply costs; and an allowance to cover indirect costs to the institution. Proposals for 1959 summer programs of this type must be received by the Foundation by January 5, 1959. (Earlier notification of the fact that a proposal is in preparation will facilitate consideration.) Fifteen copies of each proposal will be required. The original should bear the signature of the proposed project director and an official authorized to sign for his insti-

tution. All copies should indicate the names and titles of those who have signed the original of the proposal. Use of standard size duplicating paper is suggested, and each copy should be stapled at the upper left corner, not bound. Proposals should be addressed to the Special Projects in Science Education Section, Division of Scientific Personnel and Education, National Science Foundation, Washington 25, D. C. Receipt will be acknowledged.

NATIONAL SCIENCE FOUNDATION ANNOUNCES GRANTS FOR ACADEMIC-YEAR INSTITUTES FOR HIGH SCHOOL TEACHERS OF SCIENCE AND MATHEMATICS. The National Science Foundation announced the award of grants totaling over $8,600,000 to 32 colleges and universities in support of Academic-Year Institutes designed to help high school science and mathematics teachers improve their subject matter knowledge. An estimated 1,500 high school science and mathematics teachers will be enrolled in the Institutes in the 1959-60 academic year. Each teacher will pursue a program of study in the sciences and mathematics planned especially for him and conducted by leaders noted not only for competence in their fields but also for skill in presentation. The grants will provide stipends of $3,000 each to approximately 50 teachers in each Institute. Allowances for dependents and travel will also be provided. Certain Institutes will provide an additional summer training program to make it more readily possible for teachers to fulfill the requirements for a graduate degree. Supplementary allowances will be provided for teachers participating in this extended program.

The following list gives the institutions receiving grants for Institutes with exclusively mathematical programs, and also indicates the directors of each Institute. Applications by high school teachers for acceptance in the Institutes should be sent to the directors. All selections will be made by the host institution, NOT by the National Science Foundation.

Boston College	Rev. S. Bezuszka, S. J., Department of Mathematics, Chestnut Hill 67, Massachusetts
Illinois, University of	Prof. J. Landin, Department of Mathematics, Urbana, Illinois
Kansas, University of	Prof. G. B. Price, Department of Mathematics, Lawrence, Kansas
Louisiana State University	Professor H. T. Karnes, Department of Mathematics, University Station, Baton Rouge 3, Louisiana

Minnesota, University of	Prof. C. Hatfield, Department of Mathematics, S. L. A., 119 Folwell Hall, Minneapolis, Minnesota
Notre Dame, University of	Prof. A. E. Ross, Department of Mathematics, Notre Dame, Indiana
San Diego State College	Prof. J. E. Eagle, Mathematics Department, San Diego 15, California

SUMMER INSTITUTES FOR COLLEGE AND HIGH SCHOOL TEACHERS SPONSORED IN 1959 BY THE NATIONAL SCIENCE FOUNDATION. Following are the addresses of the 1959 Summer Institutes on Mathematical Topics for college teachers and high school teachers to be sponsored by the National Science Foundation. Application blanks may be obtained from the directors named in the list. Applications should be received by February 16, 1959.

1. Addresses of 1959 Summer Institutes on mathematics and statistics for college teachers, including junior college teachers in most cases.

Professor C. B. Tompkins, Department of Mathematics, University of California, Los Angeles 24, California.

Professor J. J. Gergen, Department of Mathematics, Duke University, Durham, North Carolina.

Dr. J. A. Greenlee, Division of Science, Iowa State College, Ames, Iowa.

Dr. Frederick H. Young, Department of Mathematics, Montana State University, Missoula, Montana.

Dr. F. E. McVay, Department of Experimental Statistics, North Carolina State College, Raleigh, North Carolina.

Dr. Carl E. Marshall, Statistical Laboratory, Oklahoma State University, Stillwater, Oklahoma.

Dr. D. Victor Steed, Department of Mathematics, University of Southern California, Los Angeles 7, California.

Dr. W. Norman Smith, Department of Mathematics, University of Wyoming, Laramie, Wyoming.

Dr. Edward C. Bryant, Department of Statistics, University of Wyoming, Laramie, Wyoming.

Professor R. N. Little, Department of Physics, The University of Texas, Austin 12, Texas.

2. Addresses of 1959 Summer Institutes on mathematics for college and high school teachers. (Those marked (jc) are for high school teachers and junior college teachers.)

Dr. Charles F. Brumfiel, Department of Mathematics, Ball State Teachers College, Muncie, Indiana.

Professor Frantisek Wolf, Engineering and Sciences Extension, Room 100, Building T-11, University of California, Berkeley 4, California.

Professor Charles T. Bumer, Department of Mathematics, Clark University, Worcester 10, Massachusetts. (jc)

Professor G. Baley Price, Department of Mathematics, University of Kansas, Lawrence, Kansas.

Professor James H. Zant, Department of Mathematics, Oklahoma State University, Stillwater, Oklahoma.

Mr. J. A. Farrington, Office of the Dean of the Faculty, 122 Pyne Administration Building, Princeton University, Princeton, New Jersey.

Professor Burrowes Hunt, Department of Mathematics, Reed College, Portland 2, Oregon. (jc)

Dr. W. L. Williams, Department of Mathematics, University of South Carolina, Columbia, South Carolina.

A SYMPOSIUM ON MATHEMATICAL PROGRAMMING will be held by the RAND Corporation, Santa Monica, California, on March 16-20, 1959. The Symposium will be under the general direction of George Dantzig. Among the topics to be discussed will be Problem Formulation, Solution of Large Linear Programs, Nonlinear-programming Methods, Discrete Programming, Network Flow and Distribution Problems, and Programming under Uncertainty. Papers are invited on these and related topics, both in the field of applications (March 16-17) and in the field of theory (March 18-20). The Symposium will be unclassified and there will be no charge for attendance. Abstracts of about 100 words should be submitted before January 16, 1959. Please send all abstracts, inquiries, and requests for registration forms to Dr. Philip Wolfe, The RAND Corporation, 1700 Main Street, Santa Monica, California.

THE FOURTH U. S. NATIONAL CONGRESS OF APPLIED MECHANICS will be held on the Berkeley campus of the University of California during June 18-21, 1962. Research workers in the theoretical and applied mechanics of solids and fluids are invited to submit papers for consideration by the Editorial Committee. Further announcements concerning the preparation of papers and deadlines for submission will be made as the Congress draws nearer.

The members of the organizing committee on the Berkeley campus are: Professor W. Goldsmith, Secretary; Professor E. V. Laitone, Treasurer; Professor R. M. Rosenberg, Chairman of the Editorial Committee; and Professor W. W. Soroka, General Chairman.

Inquiries regarding the Congress should be addressed to Professor W. Goldsmith, Secretary, Division of Mechanics and Design, University of California, Berkeley 4, California.

THE PROPOSED SUMMER SEMINAR IN APPLIED MATHEMATICS OF THE AMERICAN MATHEMATICAL SOCIETY scheduled for the summer of 1959 has been postponed to the summer of 1960. The topic of this Summer Seminar is "Modern Physical Theories and Associated Mathematical Developments".

DR. ROBERT D. HUNTOON NAMED DEPUTY DIRECTOR OF NATIONAL BUREAU OF STANDARDS. Dr. Allen V. Astin, Director of the National Bureau of Standards, U. S. Department of Commerce, has announced the appointment of Dr. Robert D. Huntoon to the newly created position of Deputy Director. In this post, Dr. Huntoon will serve as alternate to the Director in external matters, and will exercise day-to-day direction and review of Bureau programs, working through the Associate Directors for Engineering, Chemistry, the Boulder (Colorado) Laboratories, Planning, and Administration. He will continue to serve as Associate Director for Physics.

Dr. Huntoon joined the Bureau staff in 1941, as one of the principal scientists concerned with the development of the radio proximity fuze, considered by many to be second in importance only to the atomic bomb among World War II scientific achievements. Since then he has been, at various times, Chief of the Electronics Division, Chief of the Atomic and Radiation Physics Division, Acting Chief of the Central Radio Propagation Laboratory, Coordinator of Atomic Energy Commission projects at the Bureau, and Director of the NBS Corona (California) Laboratories, which are now operated by the Navy Bureau of Ordnance. In 1953, Dr. Huntoon was appointed Associate Director for Physics.

NEW CHIEF SCIENTIST OF THE OFFICE OF ORDNANCE RESEARCH. It has been announced that Dr. John W. Dawson, formerly Director of the Chemical Sciences Division of the Office of Ordnance Research, U. S. Army, has been appointed Chief Scientist of the Office of Ordnance Research. Dr. Dawson succeeds Dr. George Glockler.

AN ELECTRONIC COMPUTER FOR THE UNIVERSITY OF NORTH CAROLINA. The National Science Foundation has announced a grant of $500,000 to the University of North Carolina for purchase of a Univac 1105 computer for basic research purposes. This grant is the first in a series planned by the Foundation to assist in the

establishment of basic research computer centers at about 12 widely distributed regional areas throughout the United States. The University of North Carolina is planning to construct a computation laboratory, built around the computer, for basic research in the University and in other institutions in the Southeast. The Sperry Rand Corporation will contribute one-half the cost of the machine. The Census Bureau has contracted for a substantial amount of computer time during the 1960 census and other census peak periods. The State of North Carolina has given assurance of financial support for proper maintenance and full operation. The computer will be used for research problems in cosmic rays, sociometric analysis, biostatistics, econometric models, nuclear physics, and numerical analysis. Quantitative problems in the social sciences and humanities will be considered as well as in the natural sciences. The facility will also be used as a demonstration and laboratory instrument for training personnel in all areas of theoretical science concerned with the utilization of modern high-speed computers.

A UNIVAC 1105 FOR THE ILLINOIS INSTITUTE OF TECHNOLOGY. One of these electronic computing systems will be installed on the campus of the Illinois Institute of Technology this spring. It will be housed at the Armour Research Foundation. The primary uses of the Univac 1105 will relate to the Institute's educational and research activities, the Foundation's industrial and governmental research projects, and a continuing program of assistance to the U. S. Department of Commerce, Bureau of the Census, for the processing of data such as that to be obtained in the Decennial Census of 1960.

NEW FACILITIES FOR STUDIES IN MATHEMATICS AND PHYSICS AT THE CALIFORNIA INSTITUTE OF TECHNOLOGY have been assured by a grant of $1,165,700 from the Alfred P. Sloan Foundation of New York. The gift, announced there by President L. A. DuBridge, will finance construction of the new facilities within an existing building which for many years has housed Caltech's experimental high voltage laboratory. This will become a modern five-story structure (two stories below ground) with nearly 50,000 square feet of floor space, and will be renamed the Alfred P. Sloan Laboratory of Mathematics and Physics. Mr. Sloan was Chairman of the Board of General Motors Corporation from 1937 to 1956 and is now Honorary Chairman of that Board. The Sloan Laboratory, located on the south side of the campus near California Street, will provide Caltech for the first time with ample space for its fast-growing mathematics program and for expanding work in both theoretical and experimental physics. The three upper floors of the building will contain offices

for faculty members and graduate students in mathematics, as well as conference and seminar rooms, a lecture hall and a library.

The lower floors will be devoted to facilities for two experimental physics programs. To expand its research on the nuclear reactions of light elements, Caltech will install in this space a new 10-million volt Van de Graaff accelerator, which is being supplied by the Office of Naval Research at a cost of $1,000,000.

A NEW MEMOIR. Memoirs No. 30, "Flat Lorentz 3-Manifolds" by L. Auslander and L. Markus is now being printed and is expected to be available by January 1. The price is $2.00 list and $1.50 to members. The author has submitted the following description of this Memoir: "A flat Lorentz manifold is a compact differentiable manifold with a complete, indefinite metric g_{ij} having zero curvature. This Memoir determines all flat Lorentz 3-manifolds, with commutative holonomy group in the proper Lorentz group, both up to topological and up to affine type. There are infinitely many topologically distinct such manifolds, some of which bear non-countably many affinely distinct Lorentz structures. In contrast to the classical Euclidean case, the holonomy groups are not necessarily closed and the Lorentz manifolds are not all covered by toruses. An interesting infinite class of Lorentz 4-manifolds is also constructed and these correspond to finite worlds of special relativity in mathematical cosmology.

CORRECTION. REFERENCE: RUSSIAN SCIENTIFIC JOURNALS AND JOURNAL ARTICLES AVAILABLE IN ENGLISH, PAGES 537-539, OCTOBER, 1958 NOTICES. The prices for the translated Russian journal entitled "Theory of Probability and its Applications" (page 539) and available from the Society for Industrial and Applied Mathematics are incorrect. Correct prices for this journal are as follows: $18.00 per year, $9.50 per year to University and College main and departmental libraries. Add $3.00 to the above prices for foreign orders.

VISITING FOREIGN MATHEMATICIANS. The following list of visiting foreign mathematicians has been prepared by the Division of Mathematics of the National Academy of Sciences - National Research Council. This list is dated October 27, 1958. Certain errors were noted in the list as received on November 17 in the Headquarters Offices of the Society. An attempt was made to correct these insofar as information was immediately available in the membership files of the Society, but in the interests of prompt and timely publication of the list, no other effort was made to increase the accuracy of the information. The list is therefore published with the understanding that the NOTICES accepts responsibility for the accuracy of the information only to the extent that it should reflect information available in the membership files of the Society.

Name	Home Country	Host Institution	Period of visit From	To
Akizuki, Yasuo	Japan	Harvard University	Sept. 1, 1958-	Dec. 31, 1958
Altman, M.	Poland	California Institute of Technology	Oct. 1, 1958-	Sept. 30, 1959
Alvarez de Araya, Jorge	Chile	University of Washington	Sept. 1958-	Sept. 1959
Andreotti, Aldo	Italy	Institute for Advanced Study	Sept. 1958-	April 1959
Araki, Shoro	Japan	Institute for Advanced Study	Sept. 1958-	April 1959
Asplund, O. Edgar	Sweden	Institute for Advanced Study	Sept. 1958-	April 1959
Aubert, Karl E.	Norway	Institute for Advanced Study	Sept. 1958-	April 1959
Austin, M.	U. K.	University of Wisconsin	Sept. 1958-	Sept. 1959
Baayen, Pieter C.	Netherlands	University of California, Berkeley	Academic Years	1957-59
Beisicovitch, A. S.	England	University of Pennsylvania	Sept. 1958-	June 1959
Bialynicki, Andrzej	Poland	University of California, Berkeley	Academic Year	1958-59
Blakers, Albert	Australia	Princeton University	Sept. 1958-	August 1959
Bofinger, Victor	Australia	North Carolina State College	June 1957-	June 1959
Cartier, Pierre	France	Institute for Advanced Study	Sept. 1958-	April 1959
Chapman, Sydney	England	University of Michigan	1958-59	

Name	Home Country	Host Institution	Period of visit From	To
Chartres, Bruce A.	Australia	Brown University and Massachusetts Institute of Technology	Sept. 1958-	May 1959
Christian, Ulrich	Germany	Institute for Advanced Study	Sept. 1958-	April 1959
Chwe, Byoung-Song	Korea	University of California, Berkeley	Academic Years	1957-59
Corsten, Leo	Netherlands	University of North Carolina	Feb. 1958-	Dec. 1958
Craggs, James W.	U. K.	Brown University	Sept. 1958-	Sept. 1959
Drazin, Philip G.	England	Massachusetts Institute of Technology	Sept. 16, 1958-	June 15, 1959
Duguid, A. M.	Australia	Brown University	Aug. 15, 1958-	May 31, 1959
Flor, Peter	Austria	Duke University	July 1958-	June 1959
Foguel, Shaul R.	Israel.	University of California, Berkeley	Sept. 1958-	June 1959
Fuhrken, Gebhard	Germany	University of California, Berkeley	Academic Years	1957-59
Garding, Lars	Sweden	Institute for Advanced Study	Sept. 1958-	Dec. 1958
Gautschi, Walter	Switzerland	American University and National Bureau of Standards	Sept. 26, 1956-	Sept. 1958
Grauert, Hans	Germany	Institute for Advanced	Sept. 1958-	April 1959
Grewe, Rudolf	Spain	University of California, Berkeley	Academic Years	1957-59
Gribben, Ronald J.	England	Massachusetts Institute of Technology	Sept. 16, 1958-	June 15, 1959
Grothendieck, Alexandre	France	Institut Henri Poincare	Sept. 22, 1958-	Jan. 31, 1959
Grunbaum, Branko	Israel	Institute for Advanced Study	Sept. 1958-	April 1959
Ha, Kwang Chul	Korea	University of North Carolina	Sept. 15, 1958-	June 30, 1959
Hammersley, J. M.	Great Britain	University of Illinois	1st. Semester	1958-59

Name	Home Country	Host Institution	Period of visit From	To
Hano, Jun-ichi	Japan	University of Chicago	Sept. 1, 1958-	Aug. 31, 1959
Helmberg, Gilbert M.	Austria	Institute of Int. Education and Tulane University	July 1958-	Sept. 1959
Helms, Hans	Denmark	Tufts University	Sept. 1958-	June 1959
Hilton, Peter	England	Cornell University	Sept. 1958-	Sept. 1959
Hull, T. E.	Canada	California Institute of Technology	Sept. 1, 1958-	Aug. 31, 1959
Irmay, Shragga	Israel	Institute of Mathematical Sciences, New York University	Sept. 1958-	August 1959
Izumi, Shin-ichi	Japan	Yale University	Sept. 1958--	Jan. 1959
Jehne, Wolfram	Germany	Institute for Advanced Study	Sept. 1958	April 1959
Klingen, Helmut	Germany	University of California	Sept. 1958-	June 1959
Kristensen, Leif	Denmark	Yale University	August 1958-	August 1959
Krzywicki, A.	Poland	University of Kansas	Oct. 1958-	July 1959
Lacombe, Daniel	France	Institute for Advanced Study	Sept. 1958-	April 1959
Laha, Radha G.	India	Catholic University	Sept. 1957-	May 1959
Leis, Rolf	Germany	Institute of Mathematical Sciences, New York University	Sept. 1958-	August 1959
Leray, Jean	France	Institute for Advanced Study	Sept. 1958-	Dec. 1958
Levy, Azriel	Israel	Massachusetts Institute of Technology	Sept. 16, 1958-	June 15, 1959
Lions, Louis J.	France	University of Maryland	Sept. 16, 1958-	Dec. 31, 1958
Mardesic, Sibe	Jugoslavia	Institute for Advanced Study	Sept. 1958-	April 1959
Matusita, Kameo	Japan	Princeton University	Academic Year	1958-59

Name	Home Country	Host Institution	Period of visit From	To
Mitchell, A. R.	Scotland	California Institute of Technology	Mar. 1, 1959-	Sept. 30, 1959
Morikawa, Hisasi	Japan	Institute for Advanced Study	Sept. 1958-	April 1959
Morland, L. W.	England	Brown University	Sept. 1, 1958-	May 31, 1959
Mostowski, Andrzej	Poland	University of California, Berkeley	Sept. 1958-	June 1959
Muki, R.	Japan	Brown University	Sept. 15, 1958-	June 30, 1959
Munn, W. D.	Scotland	Tulane University	Sept. 1958-	Sept. 1960
Nago, Hirosi	Japan	University of Michigan	1958-	1959
Nagata, Masayoshi	Japan	Harvard University	Sept. 1, 1958-	June 30, 1959
Nakaoka, Minoru	Japan	Institute for Advanced Study	Sept. 1958-	April 1959
Nannini, Amos	Italy	University of Minnesota	Sept. 1958-	June 1959
Nevanlinna, Rolf	Switzerland	Institute for Advanced Study	Sept. 1958	Dec. 1958
Noguchi, Hiroshi	Japan	University of Michigan	Sept. 1958-	Feb. 1959
Obata, Morio	Japan	University of Illinois	Sept. 1958-	Sept. 1959
Odeh, Farouk J. S.	Jordan	University of California, Berkeley	Academic Years	1956-59
Ogawa, Junjiro	Japan	University of North Carolina	August 1958-	June 30, 1959
Ohtsuka, M.	Japan	University of Kansas	March 1959-	May 1959
Oikawa, Kotaro	Japan	University of California, Los Angeles	July 1, 1958-	June 30, 1959
O'Keeffe, Jeremiah	Ireland	Institute of Mathematical Sciences, New York University	Sept. 1958-	August 1959
Okubo, Tanjiro	Japan	State College of Washington	Sept. 16, 1958-	June 10, 1959
Ostrowski, Alexander M.	Switzerland	American University and National Bureau of Standards	Sept. 2, 1958-	Feb. 1959

Name	Home Country	Host Institution	Period of visit From	To
Pedersen, Flemming Per	Denmark	University of Southern California	July 1958-	July 1959
Pleijel, A.	Sweden	University of Kansas	March 1959-	May 1959
Plis, Andrzej	Poland	Institute of Mathematical Sciences, New York University	Sept. 1958-	August 1959
Proudman, Ian	U. K.	California Institute of Technology	Sept. 1958-	Sept. 1959
Reuter, Gerd Edward	U. K.	Yale University	Sept. 1958-	April 1959
Riesz, Marcel	Sweden	University of Maryland	Sept. 1, 1958-	Dec. 31, 1958
Rohrl, Helmut	Germany	University of Chicago	Oct. 1, 1958-	June 30, 1958
Rutovitz, Denis	So. Africa	University of California, Berkeley	Sept. 1958-	June 1959
Sabharwal, Ranjit Singh	India	University of California, Berkeley	Academic Year	1958-59
Sabidussi, Gert O.	Austria	Tulane University	Sept. 1955-	Sept. 1959
Sakurai, Akira	Japan	Institute of Mathematical Sciences, New York University	Sept. 1958-	August 1959
Satake, Ichiro	Japan	Institute for Advanced Study	Sept. 1958-	April 1959
Schaefer, Helmut	W. Germany	State College of Washington	Mar. 19, 1958-	July 1959
Schmidt, Palle Frosig	Denmark	University of Minnesota	Sept. 1958-	Sept. 1959
Schopf, Andreas	Switzerland	American University and National Bureau of Standards	Oct. 1957-	June 1959
Scriba, J. Cristoph	Germany	University of Massachusetts	Aug. 1958-	Sept. 1959
Shah, S. M.	India	University of Wisconsin	Fall Semester I	1958-59
Shirmura, Goro	Japan	Institute for Advanced	Sept. 1958-	April 1959
Shisha, Oved	Israel	Technion-Israel Institute	Sept. 1, 1958-	June 30, 1959

Name	Home Country	Host Institution	Period of visit From	To
Sikorski, Roman	Poland	Institute for Advanced Study	Sept. 1958-	Dec. 1958
Sioson, Frederico M.	Philippines	University of California, Berkeley	Academic Years	1956-59
Skovgaard, H.	Denmark	California Institute of Technology	Jan. 1, 1959-	June 30, 1960
Stoll, Wilhelm F.	Germany	Institute for Advanced Study	Sept. 1958-	April 1959
Szarski, J.	Poland	University of Kansas	Oct. 1958-	July 1959
Verma, G. R.	India	Institute of Mathematical Sciences, New York University	Sept. 1958-	August 1959
Vesentini, Edoardo	Italy	Institute for Advanced Study	Sept. 1958-	April 1959
Vogel, Walter	Germany	University of Chicago	Oct. 1958-	June 1959
Wallace, David	U. K.	Princeton University	Sept. 1958-	Sept. 1959
Walter, Wolfgang	Germany	University of Maryland	Sept. 1, 1958-	June 30, 1959
Watson, Geoffrey	Australia	Princeton University	Sept. 1958-	June 1959
Wong, Yung-Chow	Hong Kong	Institute for Advanced Study, Sept.-Dec. 1958; University of Chicago, Feb. 1-June 30, 1959	Sept. 1958-	June 30, 1959
Woods, A. C.	England	Tulane University	Sept. 1957-	Sept. 1959
Yasuhara, Mitsuru	Japan	University of California, Berkeley	Academic Years	1957-59
Yevdjevich, V.M.	Yugoslavia	American University and National Bureau of Standards	Feb. 1958-	June 1959
Young, Eutiquio	Philippines	University of Maryland	Sept. 1958-	Sept. 1959
Zadunaisky, Pedro	Argentina	Princeton University	1957-1959	
Zassenhaus, H. J.	Canada	California Institute of Technology	Sept. 1, 1958-	Aug. 31, 1959
Zeeman, Erik C.	England	University of California, Berkeley	Sept. 1958-	Jan. 1959

LETTERS TO THE EDITOR

Editor's Note: Authors of letters to be published in this department of the NOTICES are asked please to limit themselves to 1,000 words. Longer letters will be published only by direction of the Council.

Editor, the NOTICES.

The following explanation of a point in the editorial policy of Mathematical Reviews as of 1959, may serve to prevent misunderstanding.

Mathematical Reviews attempts to review all mathematical research papers published as part of the standard literature. Progress reports and the like, in mimeographed or similar forms, will not ordinarily be reviewed, although exceptions may be made in cases where the material will presumably not become available in any other form. Such reports are now much used, especially in this country, and are undoubtedly important to the progress of mathematics. But the same is true of abstracts, colloquium talks, and personal letters, and these are not reviewed as such. The assumption is that results communicated in these ways will eventually appear also in some standard published form, which will then be reviewed.

THE EDITORS
MATHEMATICAL REVIEWS

MEMORANDA TO MEMBERS

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Brown University, Providence 12, Rhode Island
Bryn Mawr College, Bryn Mawr, Pennsylvania
University of Buffalo, Buffalo 14, New York

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University of California, Berkeley 4, California
University of California, Davis, California
University of California, Los Angeles 24, California
University of California, Riverside, California
Carnegie Institute of Technology, Pittsburgh 13, Pennsylvania
Case Institute of Technology, Cleveland 6, Ohio
Catholic University of America, Washington 17, D. C.
University of Chicago, Chicago 37, Illinois
University of Cincinnati, Cincinnati 21, Ohio
City College, New York 31, New York
University of Colorado, Boulder, Colorado
Columbia University, New York 27, New York

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Cornell University, Ithaca, New York

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University of Delaware, Newark, Delaware
De Paul University, Chicago 14, Illinois
University of Detroit, Detroit 1, Michigan
Duke University, Durham, North Carolina
Duquesne University, Pittsburgh 19, Pennsylvania

Emory University, Emory University, Georgia

Florida State University, Tallahassee, Florida
University of Florida, Gainesville, Florida

Georgetown University, Washington 7, D. C.
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Gettysburg College, Gettysburg, Pennsylvania
Goucher College, Towson, Maryland
Grinnell College, Grinnell, Iowa

Harpur College, State University of New York, Endicott New York
Harvard University, Cambridge 38, Massachusetts
Haverford College, Haverford, Pennsylvania
College of the Holy Cross, Worcester 3, Massachusetts
University of Houston, Houston 4, Texas

Illinois Institute of Technology, Chicago 16, Illinois
University of Illinois, Urbana, Illinois
Immaculata College, Immaculata, Pennsylvania
Indiana University, Bloomington, Indiana
Institute for Advanced Study, Princeton, New Jersey
Iowa State College of Agriculture and Mechanic Arts, Ames, Iowa
The State University of Iowa, Iowa City, Iowa

The Johns Hopkins University, Baltimore 18, Maryland

University of Kansas, Lawrence, Kansas
University of Kentucky, Lexington 29, Kentucky
Kenyon College, Gambier, Ohio

Lehigh University, Bethlehem, Pennsylvania
Louisiana State University and Agricultural and Mechanical College, Baton Rouge 3, Louisiana

McGill University, Montreal, Quebec, Canada
University of Manitoba, Winnipeg, Manitoba, Canada
University of Maryland, College Park, Maryland
Massachusetts Institute of Technology, Cambridge 39, Massachusetts

University of Massachusetts, Amherst, Massachusetts
Mathematical Association of America, Buffalo 14, New York
University of Miami, Coral Gables, Florida
University of Michigan, Ann Arbor, Michigan
Michigan State University of Agriculture and Applied Science, East Lansing, Michigan
University of Minnesota, Minneapolis 14, Minnesota
Mississippi State College, State College, Mississippi
University of Mississippi, University, Mississippi
University of Missouri, Columbia, Missouri
Montana State College, Bozeman, Montana
Montana State University, Missoula, Montana
Mount Holyoke College, South Hadley, Massachusetts

University of Nebraska, Lincoln 8, Nebraska
University of New Hampshire, Durham, New Hampshire
New Mexico College of Agriculture and Mechanic Arts, State College, New Mexico
University of New Mexico, Albuquerque, New Mexico
New York University, New York 12, New York
University of North Carolina, Chapel Hill, North Carolina
North Texas State College, Denton, Texas
Northwestern University, Evanston, Illinois
University of Notre Dame, Notre Dame, Indiana

Oberlin College, Oberlin, Ohio
Ohio State University, Columbus 10, Ohio
Oklahoma State University, Stillwater, Oklahoma
University of Oklahoma, Norman, Oklahoma
Oregon State College, Corvallis, Oregon
University of Oregon, Eugene, Oregon

Pennsylvania State University, University Park, Pennsylvania
University of Pennsylvania, Philadelphia 4, Pennsylvania
University of Pittsburgh, Pittsburgh 13, Pennsylvania
Pomona College, Claremont, California
Portland State College, Portland, Oregon
Princeton University, Princeton, New Jersey
Purdue University, Lafayette, Indiana

Queens College, Flushing 67, New York

Randolph-Macon Woman's College, Lynchburg, Virginia
Reed College, Portland, Oregon
University of Rhode Island, Kingston, Rhode Island
Rice Institute, Houston 1, Texas
University of Rochester, Rochester 3, New York
Rutgers, The State University, New Brunswick, New Jersey

The College of St. Thomas, St. Paul 5, Minnesota
University of Santa Clara, Santa Clara, California
University of Saskatchewan, Saskatoon, Saskatchewan, Canada
Seattle University, Seattle 22, Washington
Smith College, Northampton, Massachusetts
South Dakota State College of Agriculture and Mechanic Arts, Brookings, South Dakota
University of Southern California, Los Angeles 7, California
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ABSTRACTS OF CONTRIBUTED PAPERS

THE OCTOBER MEETING IN PRINCETON, NEW JERSEY

October 25, 1958

549-58. V. E. Beneš: Type I counters with arbitrary particle arrivals.

The dead time of a Type I counter is not affected by particles arriving during that dead time. Let x_t be the dead time remaining at time t on a Type I counter; $x_t = 0$ means the counter is ready to count again. The probability measure for x_t is determined by : (i) dead times form a renewal process under the distribution F(.), with transform $F^*(\tau)$; (ii) particles arrive in an arbitrary stochastic process $\Lambda(t)$ = number of particles arriving in (0,t), independent of the dead times. Let $\mathcal{F}_t$ be the Borel field generated by the $\Lambda(s)$ for $s \leq t$, let $P(t,w) = \Pr\{x_t \leq w | \mathcal{F}_t\}$, and let $P^*(\tau)$ = Laplace transform of $\Pr\{x_t = 0\}$. We prove: (i) P(t,w) satisfies a (stochastic) Kolmogorov equation analogous to one holding for $\Pr\{x_t \leq w\}$ in the Markov case of Poisson arrivals; (ii) the expected number of particles recorded in (0,t) has Laplace-Stieltjes transform $[E\{\exp(-\tau x_0)\} - \tau P^*(\tau)]/[1 - F^*(\tau)]$; (iii) under weak hypotheses on $\Lambda(t)$, the counting rate is asymptotically $[1 - \Pr\{x_\infty = 0\}]/(\text{mean dead time})$, = maximum possible counting rate times fraction lost. (Received September 17, 1958.)

549-59. Paul Fife and Eugene Isaacson: Boundary-layer phenomena in the nonlinear bending of thin circular plates and shells. I.

Existence is proven, and an asymptotic expansion given, for solutions to von Karman's equations for the deflection of a thin circular plate under a normal load, with the various edge conditions treated by Bromberg (Comm. Pure Appl. Math. vol. 9, no. 4 (1956)). The expansion includes interior and boundary-layer terms, and under certain requirements on the edge conditions (such as: pinned or tension prescribed, and clamped or free to rotate), is proven to be asymptotic to first order in the thickness. The results extend also to spherical shells subjected to internal pressure. (Received September 12, 1958.)

549-60. A. W. Tucker: Column-dependence relations on matrices.

Let an $m \times n$ matrix C and a $\mu \times \nu$ matrix Γ, $\mu \geq m$, $\nu = m + n$, be such that $\Gamma_2 = \Gamma_1 C$ for some Γ_1 and Γ_2 formed from Γ's columns in any order, m linearly independent columns in Γ_1 and the remaining n in Γ_2. Call this relation "C is a core (column-resolvent) of Γ." If C and D are cores of Γ,

and C is a core of Δ, then D is a core of Δ. Hence, (1) "Γ and Δ have a common core" and (2) "C and D are cores of a common matrix" are equivalence relations An infinite equivalence class $\{\Gamma\}$ arises from (1), a finite one $\{C\}$ from (2). Equivalence (1) is usual row-equivalence with two additional elementary operations: interchange of any two columns, deletion or insertion of a row of zeros. A reduced matrix in $\{\Gamma\}$ is [I,C], formed by placing an identity matrix beside any core of Γ. Equivalence (2) is the combinatorial equivalence introduced by the author. (Received September 19, 1958.)

549-61. J. L. Walsh: Solution of the Dirichlet problem for the ellipse by interpolating harmonic polynomials.

If assigned boundary values satisfy a Lipschitz condition of some positive order with respect to arc length, a sequence of harmonic polynomials uniformly convergent in the closed interior of the ellipse to the solution of the corresponding Dirichlet problem is found by interpolation on the boundary in points equally distributed with respect to the conjugate of Green's function for the exterior of the ellipse with pole at infinity. (Received October 25, 1958.)

550-26. Smbat Abian and A. B. Brown: On the solution of an nth order implicit differential equation.

The authors give an existence and uniqueness proof and an explicit method of solving an implicit nth order ordinary differential equation $f[x,y,y^{(1)},\ldots,y^{(n)}] = 0$ without solving explicitly for the nth derivative. The hypotheses used are weaker than those ordinarily imposed. In particular, a set of initial values satisfying the given equation is not required. A function $F[x,y,\ldots,y^{(n)}] \equiv y^{(n)} - kf[x,y,\ldots,y^{(n)}]$ is introduced, where k is a constant. Functions $Y_{m+1}(x;n) \equiv F[x,Y_m(x;0), Y_m(x;1),\ldots,Y_m(x;n)]$ and $Y_m(x;n-j) \equiv b_{n-j} + \int_a^x Y_m(t;n-j+1)dt$, $j = 1,2,\ldots, n$, are introduced, and the solution of the given differential equation is obtained as $Y(x) = \lim_{m\to\infty} Y_m(x;0)$. Four appraisals of the remainder error after the mth stage of approximation are given, two of which are valid regardless of errors in computation through the (m - 1)st stage of approximation. (Received October 8, 1958.)

550-27. V. E. Beneš: Combinatory methods in the general theory of queues.

We consider a single-server queue, with service in order of arrival, and no defections; we make no assumptions of independence or stationarity, and use no special distributions. The queue transforms a stochastic process K(t) of service-times and arrival epochs into a process W(t) of waiting-times, $t \geq 0$. There is a corresponding functional F which gives the distribution of W(t) in terms of distributions associated with K(u), $u \leq t$. The form of F, previously discovered by the author by an involved argument using stochastic Kolmogorov equations, is established by means of some simple, essentially combinatory identities. (Received October 7, 1958.)

550-28. Nathaniel Coburn and R. S. B. Ong: General and transverse waves in hydro-magnetic flows.

In this paper, two problems are considered for the steady flow of an infinitely conductive nonviscous compressible gas in a magnetic field: (1) properties of the characteristic manifolds (general waves); (2) determination of

necessary and sufficient conditions for the existence of transverse and longitudinal waves and properties of transverse waves. By introducing a local coordinate system with coordinate axes along the velocity and magnetic field vectors, the cross-sections of the characteristic cone of normals by planes parallel to the velocity or magnetic vectors are shown to be fourth-degree curves which depend upon three scalars: K^2, the ratio of the magnetic to the mechanical energy; M, the Mach number; U, the cosine of the angle between the velocity and magnetic field vectors. If U vanishes, and K is not zero, the cross-section becomes a biquadratic curve; the distance from this curve to one of its asymptotes determines the projection of the velocity on the corresponding normal. By studying the discontinuities in the derivatives of density, pressure, velocity, and the magnetic field vector, it is shown that for transverse waves, both the magnetic field and velocity vectors are uniquely determined by the normal and bicharacteristic directions. (Received October 27, 1958.)

550-29. R. L. Vaught: Homogeneous universal models of complete theories.

Let T be a complete (first order with $=$) theory, having infinite models. Let ϕ_n ($n = 0,1,\ldots$) be a list of all formulas of T with free variables $v_0,\ldots,v_{k_n}$. Let K* be the class of all systems $\langle A,R_0,\ldots,R_m; S_0,\ldots,S_n,\ldots\rangle$ such that $\mathcal{A} = \langle A,R_0,\ldots,R_m\rangle$ is a model of T and $S_n = \{\langle a_0,\ldots,a_{k_n}\rangle / \models_{\mathcal{A}} \phi_n[a_0,\ldots,a_{k_n}]\}$. Theorem 1 of the author's Abstract 550-1 Notices Amer. Math. Soc. vol. 6 (1958) p. 671, is obtained from Jónsson's result quoted there and: Theorem 1. K* fulfills conditions I-VI_1 of Jónsson. Assume below the generalized continuum hypothesis. From a result announced in Jónsson's Abstract 550-30 (in this issue of Notices) together with Theorem 1, follows: Theorem 2. If $\alpha = \beta + 1$, then there is one and, up to isomorphism, only one model $\mathcal{A}$ of T of power $\aleph_\alpha$ such that (i) $\mathcal{A}$ is an elementary (= arithmetical) extension (cf. Tarski, Vaught, Compositio Math. vol. 13 (1957) pp. 81-102) of an isomorph of every model of T of power $\leqq \aleph_\alpha$, and (ii) whenever $\mathcal{A}$ is an elementary extension of each of $\mathcal{B}$ and $\mathcal{C}$, having power $< \aleph_\alpha$, then any isomorphism between $\mathcal{B}$ and $\mathcal{C}$ can be extended to an automorphism of $\mathcal{A}$. Theorem 3. Suppose T is $\aleph_{\beta+1}$-categorical for each $\beta < \alpha$, and α is a limit ordinal ($\neq 0$). Then T is $\aleph_\alpha$-categorical and, for any model $\mathcal{A}$ of T of power $\aleph_\alpha$, (ii) holds. (Received October 6, 1958.)

550-30. Bjarni Jonsson: Homogeneous universal relational systems.

For terminology see the author's: Universal relational systems, Math. Scand. vol. 4 (1956) pp. 193-208. Given a class K of relational systems, a relational system $\mathcal{A} \in K$ is called K homogeneous if, for any subsystem $\mathcal{B} \in K$ of $\mathcal{A}$ with cardinality less than the cardinality of $\mathcal{A}$, every isomorphism of $\mathcal{B}$ into $\mathcal{A}$ can be extended to an automorphism of $\mathcal{A}$. Result: If the Generalized Continuum Hypothesis holds, if K is a class of relational systems which satisfies the conditions I-V and VI, (loc. cit. p. 195), and if α is a nonlimit ordinal, then there exists one, and up to isomorphism only one, K homogeneous ($\aleph_\alpha$,K) universal relational system. A concept related to our homogeneous systems was considered by R. Fraïssé. See, e. g. his Sur l'extension aux relations de quelques propriétés des orders, Ann. Sci. Ecole Norm. Sup. where he proves in effect the existence of K homogeneous ($\aleph_0$,K) universal systems for certain classes K of relational systems. (Received October 3, 1958.)

550-31. H. J. Keisler: The theory of models with generalized atomic formulas, II.

Using methods from I, theorems are proved concerning the classes $Q_1, Q_2, Q_3, \ldots$, of universal, UE, UEU,... sentences respectively, and concerning direct limits. A finite sequence $\mathfrak{A}_0, \mathfrak{A}_1, \ldots, \mathfrak{A}_n$ of systems is an n-sandwich iff $\mathfrak{A}_{i+2}$ is an arithmetical extension of $\mathfrak{A}_i$ for $0 \leq i \leq n-2$, and $\mathfrak{A}_i$ is a subsystem of $\mathfrak{A}_{i+1}$ for $0 \leq i \leq n-1$. Theorem: Suppose that either $K \in AC_\Delta$, or K is the class of arithmetical extensions of a system $\mathfrak{A}_0$. Then every sentence of Q_n holding throughout K also holds in $\mathfrak{A}$ iff $\mathfrak{A}$ and some $\mathfrak{B} \in K$ are the first two elements of an n-sandwich. Consider the sequences $\langle \mathfrak{A}_i \mid i < \omega \rangle$, $\langle \phi_i \mid i < \omega \rangle$, the $\mathfrak{A}_i$ being mutually disjoint, and each ϕ_i being a homomorphism of $\mathfrak{A}_i$ into $\mathfrak{A}_{i+1}$. Their direct limit is defined as follows: Elements $a_i \in A_i$, $a_j \in A_j$ are equivalent iff for some $k > \max(i,j)$, $\phi_{k-1} \cdots \phi_{j+1} \phi_j a_j = \phi_{k-1} \cdots \phi_{i+1} \phi_i a_i$. A is the set of equivalence classes of $\bigcup_{i<\omega} A_i$. If $\langle a,b,c \rangle \in A^{(3)}$, $\mathfrak{A}(a,b,c)$ iff for some $i < \omega$, $\langle a_i,b_i,c_i \rangle \in A_i^{(3)}$, $\langle a_i,b_i,c_i \rangle \in a \times b \times c$, and $\mathfrak{A}_i(a_i,b_i,c_i)$. It is proved that if $K \in AC_\Delta$, then the class characterized by all UE sentences, holding in K, in which every existentially quantified variable occurs only positively, is the class of arithmetical subsystems of direct limits of models of K. (Received October 6, 1958.)

550-32. Naoki Kimura: A theorem on the semigroup of stochastic matrices.

The set of all stochastic matrices of order n forms a semigroup under the matrix multiplication. Each idempotent matrix in it determines a maximal subgroup with this matrix as the unit. We shall prove here that any maximal subgroup is isomorphic with the symmetric group of order p where $1 \leqq p \leqq n$. (Received October 9, 1958.)

550-33. E. O. A. Kreyszig: A numerical method for solving the torsion problem of a bar.

According to Saint Venant's theory the torsion problem of a bar of uniform cross section leads to the first boundary value problem of potential theory. The imaginary part of the torsion potential satisfies the condition $g(x,y) = x^2 + y^2$ on the boundary C of the cross section B of the bar. Approximate solutions of any required degree of accuracy may be obtained by using Bergman's representation of harmonic functions as an infinite series involving the functions of a complex orthonormal system. On this basis a numerical method can be developed which corresponds to a relatively simple program for high speed computers. Various auxiliary functions appearing in the procedure are independent of the boundary values of g and can be tabulated once and for ever. The program of numerical computation is simplified if B has some regularly distributed axes of symmetry. In many cases of starshaped regions B the boundary C may be represented or approximated by a Fourier polynomial. Then the integrals representing the coefficients in the abovementioned representation can be evaluated in closed form. Estimates of the error caused by that approximation of C can be derived. The method is useful also for other boundary value problems in connection with simply-connected regions. (Received May 2, 1958.)

550-34. Joachim Nitsche: Simple error bounds for eigenvalues and eigenvectors of hermitian matrices.

Let A be a $n \times n$ matrix with eigenvalues λ_i and eigenvectors x_i. Suppose n orthonormal vectors y_i have been computed which are approximations for the x_i. Denote by μ_i the Rayleigh quotients $\mu_i = y_i^T A y_i$. Author proves the following inequalities: $|\lambda_i - \mu_i| \leqq (n/2)^{1/2}\epsilon$ where $\epsilon = \mathrm{Max}_i\{y_i^T A^2 y_i - (y_i A y_i)^2\}^{1/2}$. If in addition the inequality $e(2n)^{1/2} < q, q = \mathrm{Min}_{i \neq k}|\mu_i - \mu_k|$ is satisfied then the eigenvalues λ_i of A are simple and the further inequalities $|y_i - x_i| < e[1 + e$

$(q - (n/2)^{1/2}\epsilon)^{-1}]\cdot(q - \epsilon(n/2)^{1/2})^{-1}$ hold true. The preceding inequalities are computationally particularly simple. (Received October 6, 1958.)

550-35. D. S. Scott: On the Löwenheim-Skolem theorem in weak second-order logic.

For notation see Abstract 550-6, Notices Amer. Math. Soc. vol. 6 (1958) p. 673. By a result mentioned there, if a set Σ of WS-sentences has a model of infinite power $\mathfrak{M}$, then it has also models of all infinite powers less than $\mathfrak{M}$. Tarski pointed out that for $\mathfrak{M} = \aleph_0$ and $\mathfrak{M} = 2^{\aleph_0}$ there are sets Σ which have models of power $\mathfrak{M}$ but of no higher power; he asked the question whether there were other such cardinals. The answer is affirmative. In fact, set $\aleph_{\pi(0)} = \aleph_0$, $\aleph_{\pi(\alpha+1)} = 2^{\aleph_{\pi(\alpha)}}$, and $\aleph_{\pi(\beta)} = \sum_{\xi<\beta} \aleph_{\pi(\xi)}$ for limit ordinals β. Then the following holds: If $\mathfrak{M} = \aleph_{\pi(\gamma)}$, where γ is the type of a well-ordering relation elementarily definable in Peano's arithmetic, then there is a set Σ of WS-sentences having models of power $\mathfrak{M}$ but of no higher power. To obtain the desired set Σ for a given ordinal γ, proceed as follows: Let U_0 be the empty set, $U_{\alpha+1}$ be the set of all subsets of U_α, and $U_\beta = \bigcup_{\xi<\beta} U_\xi$ for limit ordinals β. Then for Σ take the set of all WS-sentences containing the membership symbol ϵ as the only non-logical constant which are true if the first-order variables are assumed to range over elements of $U_{\omega+\gamma}$. (Received October 3, 1958.)

550-36. D. S. Scott and Alfred Tarski: Extension principles for algebraically closed fields.

For notation see Abstract 550-6, Notices Amer. Math. Soc. vol. 6 (1958) p. 673. $\mathfrak{A}$ and $\mathfrak{B}$ being any algebraically closed fields with the same characteristic, the following two results hold: (I) $\mathfrak{A}$ and $\mathfrak{B}$ are always F-equivalent; (II) $\mathfrak{A}$ and $\mathfrak{B}$ are WS-equivalent iff either both of them have the same finite transcendence degree or else each of them has infinite transcendence degree over the prime field. (I) is known from the literature; see, e.g., Bull. Amer. Math. Soc. vol. 55, Abstract 77. (II) is closely related to results mentioned in Abstract 544-44 Notices Amer. Math. Soc. vol. 5 (1958) p. 221 and easily follows from results in Abstract 550-6 Notices Amer. Math. Soc. vol. 6 (1958) p. 673. By (I) every F-sentence (formulated in terms of the theory of fields) which has been established for complex numbers automatically extends to all algebraically closed fields with characteristic 0; by (II) every WS-sentence so established

extends to all those algebraically closed fields with characteristic 0 which have infinite transcendence degree over the field of rationals. (I) extends to a comprehensive class of WS-sentences, but no simple characterization of all WS-sentences to which (I) can be extended is known. Presumably all known theorems of algebraic geometry can be formulated as WS-sentences. (Received October 3, 1958.)

550-37. G. B. Seligman: On the group of invariant automorphisms of a Lie algebra.

If L is a simple Lie algebra of classical type, the group I of invariant automorphisms is a normal subgroup of the full automorphism group A (for terminology, see J. Math. Mech. vol. 6 (1957) pp. 519-558). If the base field F is algebraically closed, then $I = A$ unless L is of type A_n ($n \geq 2$), D_n ($n \geq 4$), E_6. In these cases, an upper bound (which is in fact attained) for the index of I in A is determined. Substantial portions of the results extend to the case where F is not algebraically closed. Except for the final statements about specific cases, the techniques do not use the classification of the algebras. (Received September 15, 1958.)

550-38. A. H. Stone: Universal spaces for some metrisable uniformities.

It is shown that several types of metrisable uniformities, definable in terms of simple families of coverings, can be characterized by their uniform equivalence to subspaces of certain "universal" spaces of the same type. This holds in some familiar cases, and also in the following ones, where the "universal" spaces U_1, U_2, depend only on the least cardinal of a dense set in the space R. (1) If a metrisable uniformity on R has a basis of Euclidean coverings (Isbell, Pacific J. Math. vol. 8 (1958) p. 70), then it is uniformly equivalent to a subspace of the product of countably many lines, and conversely. (2) If every covering of R is uniform, R is uniformly equivalent to a closed subspace of a space U_1 with the same property, and conversely. (3) If the uniformity on R has a basis of star-countable coverings, R is uniformly equivalent to a subspace of a space U_2 with the same property, and conversely. (Received September 25, 1958.)

550-39. R. L. Vaught: Prime models and saturated models.

Consider complete theories T having infinite models. For $n = 1,2,\ldots$, form $\mathfrak{B}_n(T)$, the Boolean algebra of formulas of T whose free variables are $v_1,\ldots,v_n$, modulo equivalence in T. (These algebras were considered earlier by Ryll-Nardzewski and Ehrenfeucht in characterizing $\aleph_0$-categorical theories -- cf. Mostowski, Colloque International du C. N. R. S. 1955, p. 24, to appear.) Call $\mathfrak{A}$ a prime model of T, if every model of T is an elementary extension of an isomorph of $\mathfrak{A}$. (Compare Robinson, Complete theories, Amsterdam, 1956.) Theorem (1). T has a prime model if and only if, for each n, $\mathfrak{B}_n(T)$ is atomistic. Call $\mathfrak{A}$ a saturated model of T if (i) $\mathfrak{A}$ is denumerable and is an elementary extension of an isomorph of each denumerable model of T, and (ii) if $m \in \omega$ and $\langle a_0,\ldots,a_m\rangle$ and $\langle a'_0,\ldots,a'_m\rangle$ satisfy in $\mathfrak{A}$ the same formulas of T, then there is an automorphism f of $\mathfrak{A}$ with $f(a_i) = a'_i$, for $i \leq m$. Theorem (2). The following are equivalent: (a) T has a model fulfilling (i); (b) for each n, $\mathfrak{B}_n(T)$ has at most $\aleph_0$ prime ideals; (c) T has a saturated model. Theorem (3). T has, up to isomorphism, at most one prime model and at most one saturated model. Using a result of Ehrenfeucht (Fund. Math. vol. 14 (1957) p. 247), the following is shown: Theorem (4). If T is $2^{\aleph_0}$-categorical, then T has a prime model and a saturated model. Alternative characterizations of prime and saturated models may be given, involving the $\mathfrak{B}_n(T)$. (Received October 6, 1958.)

THE NOVEMBER MEETING IN DURHAM, NORTH CAROLINA

November 28-29, 1958

551-28. Leonard Carlitz: Some finite summation formulas of arithmetic character.

Mikola's (Publ. Math. Debrecen vol. 5 (1957) pp. 44-53) has proved the elegant formula (*) $\int_0^1 \bar{B}_r(ax)\bar{B}_r(bx)dx = (-1)^{r-1}(r!r!/(2r)!)(a,b)^r[a,b]^{-r}B_{2r}$, where $\bar{B}_r(x)$ is the Bernoulli function; more generally he has proved a formula of this type for the Hurwitz zeta-function. Recently Mordell (J. London Math. Soc. vol. 33 (1958) pp. 371-375) has proved a general formula which contains (*) as a special case. The present paper is concerned with finite analogs of these results. In particular, the sum $\sum_{r=0}^{K-1}\bar{B}_m(ar/K)\bar{B}_n(br/K)$, where $K = k[a,b]$, $m + n \geq 2$, is evaluated. For $k = \infty$, $m = n$, this result reduces to (*). (Received November 16, 1958.)

551-29. Eckford Cohen: A class of arithmetical functions of several variables, I. Applications to congruences.

The concept of even function (mod r) is extended to functions of k variables. For simplicity, let $k = 2$. A function $f_r(m,n)$ is said to be even (mod r) if it is even in the variables m and n separately, and totally even (mod r) if it is even as a function of (m,n). Trigonometric characterizations of the even and totally even functions (mod r) are obtained in terms of analogues of the Ramanujan sums. Numerous applications to simultaneous congruences are considered, in particular to the determination of the number of solutions of pairs of linear and bilinear congruences (mod r). Certain special subclasses of even functions $f_r(m,n)$ are also considered, with applications to simultaneous partitions (mod r). (Received July 31, 1958.)

551-30. Eckford Cohen: Arithmetical functions associated with arbitrary sets of integers.

Let S be an arbitrary set of positive integers. The characteristic function $\rho_S(n)$ of S is defined by $\rho_S(n) = 1$ or 0 according as $n \in S$ or $n \notin S$, while the inversion function $\mu_S(n)$ of S is defined by $\sum_{n=1}^{\infty}\mu_S(n)/n^t = \zeta^{-1}(t) \cdot \sum_{n=1}^{\infty}\rho_S(n)/n^t$. Define $N_k(x,S)$ to be the number of positive integral vectors

$\{n_1,\ldots,n_k\}$, $n_i \leq x$, such that $(n_1,\ldots,n_k) \in S$. Using the concept of inversion function, estimates for $N_k(x,S)$, $k \geq 2$, proved previously in case S is a direct factor set, are extended to arbitrary sets. The results in the general case are identical with the earlier results except for the case $t = 2$, $\mu_S(n)$ unbounded. Various properties of divisor functions, totient functions, and summatory functions are also extended to arbitrary sets S. The method of arithmetical inversion, used in the case of direct factor sets, is no longer adequate. (Received September 22, 1958.)

551-31. Eckford Cohen: Asymptotic averages in a class of arithmetical functions of two variables.

Let f(n) be an arithmetical function satisfying certain restrictions as to its order of magnitude. This paper is concerned with the average order of functions of the form f((m,n)) in the two integral variables m,n. Special cases of the main results include asymptotic formulas for the average of $\sigma_\alpha((m,n))$ and $\phi_\alpha((m,n))$, where $\sigma_\alpha(n)$ and $\phi_\alpha(n)$ denote the generalized divisor and totient functions, respectively. The results for $\sigma_\alpha((m,n))$ yield refinements of earlier estimates due to Cesàro. Of the many formulas deduced, the following is cited as typical: $3\sum_{a,b\leq x}\sigma_2((a,b)) = x^3(2\zeta(2) - \zeta(3)) + O(x^2\log x)$. The method of the paper is based on certain arithmetical identities of an elementary nature. (Received October 2, 1958.)

551-32. M. L. Curtis and M. K. Fort, Jr.: Singular homology of one-dimensional spaces.

Let X be a one-dimensional separable metric space. The authors have previously shown that the homotopy groups $\pi_k(X)$ are trivial for all $k > 1$. (Proc. Amer. Math. Soc. vol. 8 (1957) pp. 557-559). The present paper proves that the integral singular homology groups $H_k(X,Z)$ are trivial for all $k > 1$. An outline of the proof follows: (1) If Y is a subset of X, then $\pi_1(Y)$ is a subgroup of $\pi_1(X)$. (2) The fundamental group of the Menger universal one-dimensional curve is locally free; i.e. its finitely generated subgroups are free. (3) The homology groups of a locally free group are trivial in dimensions greater than one. Since X is a $K(\pi,1)$, the statements (1), (2), and (3) imply the triviality of the singular groups of X. (Received October 3, 1958.)

551-33. M. K. Fort, Jr.: One-to-one mappings onto the Cantor set.

Let X be a separable metric space such that: X is zero-dimensional, each point of X is a limit point of X, and X can be imbedded as a G_δ subset of some complete metric space. It is shown that each such space X can be mapped by a one-to-one continuous function onto the Cantor Middle-third set. In particular, a special case of this result asserts that there is a one-to-one continuous function whose domain is the set of all irrational numbers, and whose range is the Cantor set. (Received October 3, 1958.)

551-34. M. K. Fort, Jr.: Images of plane continua.

Let K be the continuum which consists of all points in the plane having polar coordinates (r,θ) for which $r = 1$, $r = 2$, or $r = (2 + e^{\theta})/(1 + e^{\theta})$. The following two theorems are the main results obtained in this paper. Theorem 1. A plane continuum which does not separate the plane cannot be mapped continuously onto K. Theorem 2. A plane continuum cannot be mapped continuously onto the dyadic solenoid. The second of these theorems generalizes the known theorem which states that the dyadic solenoid cannot be imbedded in a plane. (Received October 3, 1958.)

551-35. Frank Harary: An elementary theorem on graphs.

Let G be a graph with p points and k connected components. Let N be the number of blocks of G (a block is a maximal connected subgraph containing no cut points of itself). Let n_i be the number of components of the subgraph of G obtained on removing the i'th point. Then the following equation holds: $N - (n_1 + n_2 + \ldots + n_p) + p = k$. For trees, this specializes at once to the familiar statement that the number of points is one greater than the number of lines. The formula is easily proved by induction. (Received October 9, 1958.)

551-36. J. H. Hodges: Some determinantal equations related to representations by forms over a finite field.

Let GF(q), $q = p^n$ denote the finite field of q elements. The number of solutions U,V is found of the determinantal equation $\det(M) = \beta$, where $\beta \in GF(q)$ and M is a square matrix of order $m + t$ over GF(q) having rows $M_1 = (A,V)$, $M_2 = (U,0)$, where A is a nonsingular matrix of order m, V is an $m \times t$ matrix and U is a $t \times m$ matrix. The solution of this problem is reduced to a problem of representations by bilinear forms over GF(q) which has been previously solved

by the author (Representations by bilinear forms in a finite field, Duke Math. J. vol. 22 (1955) pp. 497-509). An analogous equation related to representations by hermitian forms over $GF(q^2)$ is also considered. The methods of the present paper are due to L. Carlitz, who has solved analogous equations for symmetric matrices (A problem involving quadratic forms in a finite field, Math. Nachr. vol. 11 (1954) pp. 135-142) and skew matrices (Representations by skew forms in a finite field, Arch. Math. vol. 5 (1954) pp. 19-31). As in Carlitz's papers, generalized Gauss sums, related to the determinantal equations, are defined and evaluated. (Received October 6, 1958.)

551-37. R. J. Koch: Ordered semigroups in partially ordered semigroups.

A local semigroup S is defined to be partially ordered if the relation $\leq$ defined by $a \leq b$ if and only if $a = bc$ is reflexive and antisymmetric. If S is a semigroup, then $\leq$ is also transitive. The following theorem is established: A locally compact connected partially ordered local semigroup S with unit contains a nondegenerate compact connected linearly ordered local subsemigroup (containing the unit). This is an extension of a theorem of Gleason [Proc. Nat. Acad. Sci. vol. 36 (1950)], who proved a similar theorem under the additional hypotheses that (1) S is a semigroup with right invariant uniform structure and (2) for any compact neighborhood U of the unit there are nets $\{x_i\}$ in S and $\{n_i\}$ (integers) such that $x_i \to e$ and $x_i^{n_i} \notin U$. It follows that a compact connected partially ordered semigroup with unit and zero contains an arc from zero to unit which is a semigroup. (Received October 6, 1958.)

551-38. R. J. Koch and I. S. Krule: Concerning the weak cutpoint ordering.

The following result gives a characterization of the admissibility of certain partial orderings on a hereditarily unicoherent continuum. Let X be such a continuum, let p be a fixed element of X, and define a partial ordering $\leq_p$ by: $a \leq_p b$ if and only if each subcontinuum of X containing p and b also contains a. The following theorem obtains: If X and $\leq_p$ are as above, then the following are equivalent: (1) $\leq_p$ is monotone (i.e., $\{x: x \leq_p a\}$ is connected for each $a \in X$); (2) there exists a monotone partial order on X with closed graph and unique minimal element p. (3) X is arc-wise connected, and for any net $\{x_\alpha\}$ in X which converges to x, the net of arcs $\{[p,x_\alpha]\}$ joining p to x_α converges to the arc $[p,x]$ joining p to x. (Received October 6, 1958.)

551-39. Sibe Mardešić: Dimension and inverse limits of compact spaces.

It is known that Hausdorff compact spaces coincide with inverse limits of inverse systems of finite polyhedra. In this paper the following question is considered: Can one characterize Hausdorff compacta of dimension $\leq n$ as inverse limits of finite polyhedra of dimension $\leq n$ (dimension is taken in the sense of open coverings)? Theorem 1. If X_λ are Hausdorff compacta forming an inverse system and dim $X_\lambda \leq n$, then the limit X satisfies dim $X \leq n$. Let ind denote the inductive dimension. Theorem 2. If X_λ are finite polyhedra forming an inverse system and dim $X_\lambda \leq 1$ ($\leq n$ and the mappings of the system are piecewise linear), then the limit X is a Hausdorff compact with ind $X \leq 1$ (ind $X \leq n$). Theorem 3. If X is a Hausdorff compact with dim $X = 1$ and ind $X > 1$, then X cannot be obtained as an inverse limit of polyhedra of dimension ≤ 1. Hausdorff compacta X, with dim $X = 1$, ind $X = 2$, have been found by A. Lunc (Dokl. Akad. Nauk SSSR vol. 66 (1949) pp. 801-803) and O. V. Lokucievskiĭ (Dokl. Akad. Nauk SSSR vol. 67 (1949) pp. 217-219). In the metrizable case the answer to the question of above is positive (with sequences replacing directed systems) as has been shown already by H. Freudenthal (Compositio Math. vol. 4 (1937) p. 229). (Received September 29, 1958.)

552-19. Smbat Abian and A. B. Brown: On the solution of simultaneous implicit equations.

The authors have used the method of successive substitutions for solving a system of p simultaneous implicit equations $f_i(x_1,\ldots,x_n,y_1,\ldots,y_p) = 0$ for the p unknown functions $y_i = Y_i(x)$. The hypotheses of the classical implicit function theorem are replaced by weaker hypotheses. In particular, the functions f_i are not required to be differentiable, and there is no requirement that a known point satisfy the given equations. Two appraisals of the remainder error at the m th stage of approximation are given, one of which is valid regardless of errors made at earlier stages of the computation. It is also proved that if the given functions f_i satisfy Lipschitz conditions in a certain subset of the x's, then the $Y_i(x)$ will also satisfy Lipschitz conditions in the same subset. (Received August 15, 1958.)

552-20. Eckford Cohen: Inversion of primitive functions (mod r) and Nagell's totient function.

If n and r are integers, $r > 0$, then a function $f(n,r)$ is called primitive (mod r) if $f(n,r) = f(\gamma(n,r))$ where $\gamma(n,r) = \gamma((n,r))$ and $\gamma(r)$ denotes the product of the distinct prime divisors of r. A simple proof of the following inversion relation is given. Let r be primitive (free of square divisors > 1) and let $f(n,r)$ be primitive (mod r), $r = r_1r_2$; then $f(n,r) = \sum_{d|r,(d,n)=1} F(d,r/d)$ $\Leftrightarrow F(r_1,r_2) = \mu(r_1)\sum_{d|r_1} f(r/d,r)\,\mu(d)$. This principle is applied to give a new proof of the author's evaluation (Duke Math. J. vol. 25 (1958)) of Nagell's totient function $\theta(n,r)$, defined to be the number of integers a (mod r) such that $(a,r) = (n - a,r) = 1$. (Received September 5, 1958.)

552-21. R. G. Bartle: On the openness and inversion of differentiable mappings.

This note is closely related to recent papers of R. and F. Nevanlinna [Ann. Acad. Sci. Fenn., nos. 185 and 245]. Let f be a map of the ball $|x| < r$ of a Banach space X into a space Y, with $f(0) = 0$. Assume (A) f is differentiable, (B) $f'(0)$ maps X onto Y, and (C) $\limsup \| f'(x) - f'(0) \| < \mu$, where μ is the radius

of the largest ball covered by $f'(0)(|x| < 1)$. Then (1) numbers ρ_x and ρ_y are found such that f sends any open set in $|x| < \rho_x$ into an open set; (2) $f(|x| < \rho_x)$ contains the ball $|y| < \rho_y$; (3) the numbers ρ_x and ρ_y cannot be increased; (4) if $f'(0)$ is one-one, then f is one-one on $|x| < \rho_x$ and its inverse is differentiable. This extends a result of L. M. Graves [Duke Math. J. vol. 17 (1950)], but our proof uses his result. This note will be published in the Ann. Acad. Sci. Fenn. Ser. A. I. (Received September 29, 1958.)

552-22. Edward Fadell: On fiber homotopy equivalence.

A. Dold (Math. Z. vol. 62 (1955) pp. 111-136) gives a very useful necessary and sufficient condition for two fiber bundles (over a common polyhedral base with locally compact fibers) to be fiber homotopy equivalent. The objective of this paper is to extend Dold's result to Hurewicz fibrations with no local compactness assumption on the fibers involved. This extension is based on a suitable fiber homotopy extension theorem and the use of quasi-topologies in certain function spaces. This extension then provides a tool for showing that certain fiber spaces and fiber bundles are fiber homotopy equivalent. As one application we show that any universal bundle over a polyhedron P, whose group is dominated by a CW-complex, is fiber homotopy equivalent to the fiber space of paths emanating from a fixed point of P. Thus Milnor's universal bundle (Ann. of Math. vol. 63 (1956) pp. 272-284) over P is fiber homotopic to this fiber space of paths. (Received October 3, 1958.)

THE ANNUAL MEETING IN PHILADELPHIA, PENNSYLVANIA

January 20-22, 1959

553-1. Martin Schechter: General boundary value problems for elliptic partial differential equations. III.

Let G be a bounded domain in E^n and let A be a properly elliptic linear operator of order $2r$ with variable complex coefficients. Consider the two sets of boundary operators $M_t = \sum_{k=1}^{2r} b_{tk} D_n^k$, $N_t u = \sum_{s=1}^{2r} \beta_{ts} \sum_{|\tau|+k \leq 2r} D^\tau D_n^{k-s} (a_{k\tau}^* u)$, where the b_{tk}, β_{ts} are complex functions defined in $\dot{G}$, D_n is the normal derivative, D^τ is any tangential derivative of order $|\tau|$, and $a_{k\tau}^*$ is the coefficient of $D^\tau D_n^k$ in A^*, the formal adjoint of A. The boundary problem (A, f, u_0, M_t) is to find a function u such that $Au = f$ in G and $M_t(u - u_0) = 0$ on $\dot{G}$ for all t. It is proved that if the M_t cover A and the b_{tk} and β_{ts} are related in a certain manner, then the solution of one of the problems (A, f, u_0, M_t), (A^*, f, u_0, N_t) exists provided the other is unique. The solution is classical provided $f, u_0, \dot{G}$, and the coefficients of A, M_t, N_t are sufficiently smooth. In the case of the Dirichlet problem, $M_t = N_t$ and they cover any properly elliptic operator. Hence the solution of the Dirichlet problem for A exists if and only if the same problem for A^* is unique. (Received August 25, 1958.)

553-2. C. L. Dolph and F. Penzlin: On the theory of a class of non-self-adjoint operators and its applications to quantum scattering theory.

Let R, S be Hermitian operators in a Hilbert space $\mathcal{H}$ such that S is positive semi-definite. Let the range of $A = R - iS$ be $\mathcal{H}$ for $\operatorname{Im} \lambda > 0$, then: (a) the resolvent operator $(A - \lambda I)^{-1}$ exists as a bounded operator for $\operatorname{Im} \lambda > 0$ and (b) the weak representation $((A - \lambda I)^{-1} \phi, \psi) = \int_{-\infty}^{\infty} dt (F(t)\phi, \psi)/(t - \lambda)$ exists for $\operatorname{Im} \lambda > 0$ where F(t) is self-adjoint and monotone nondecreasing. The proof is an extension of the Riesz-Herglotz-Nevanlinna approach to the Spectral Theorem as it is found in the book by Achiezer and Glasmann. If S is completely continuous and has a finite absolute trace (without necessarily being positive semi-definite) then the continuous spectrum of A is real and the number of principal vectors λ_p of A with $\operatorname{Im} \lambda_p \neq 0$ is at most denumerable. This generalizes the results of Livšic (Amer. Math. Soc. Translations, Ser. 2, vol. 5 1957), who considered only bounded operators A. The proof is operator theoretic in

nature and avoids use of the characteristic matrix function of Livšic. These results are then applied to the characteristic matrix operator of Livšic and to the scattering integral equations and operators of quantum mechanics after symmetrization similar to that given by Dolph and Ritt (Math. Z. vol. 65 (1956) p. 304). (Received October 3, 1958.)

553-3. S. P. Lloyd: Stochastic processes with analytic covariance.

Let stochastic process $\{x(t,\omega), t \in T\}$ have covariance $R(s,t) = E\{x(s,\omega)\overline{x(t,\omega)}\}$, $s,t \in T$, where $T = (t_0 - a, t_0 + a)$ is an open interval. Suppose the covariance is representable by a power series $R(s,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} R_{mn}(s - t_0)^m(t - t_0)^n$, convergent in the square $s \in T$, $t \in T$. Then a necessary and sufficient condition that the sample function $x(t,\omega)$ (ω fixed) be an analytic function of $t \in T$ with probability 1 is that the process be separable. Related results for stationary (wide sense) processes are the following. Let the process $\{x(t,\omega), -\infty < t < \infty\}$ be separable and stationary in the wide sense. Suppose $R(t)$ ($= R(t + \tau, \tau)$) is the restriction to real argument of an analytic function regular in a disc $|t| < 2a$. Then, with probability 1, the sample function $x(t,\omega)$ is the restriction to real argument of an analytic function regular in the strip $|\mathrm{Im}(t)| < a$. If $R(t)$ is an entire function of order ρ and type τ then, with probability 1, $x(t,\omega)$ is an entire function of growth $(\rho, 2^{\rho-1}\tau)$. (Received October 8, 1958.)

553-4. Frank Harary: A graph theoretic method for the reduction of a matrix.

Let us say that a square matrix M is reducible if and only if for some permutation matrix P the matrix PMP^{-1} is partitioned into four submatrices such that the ones in the upper left and lower right are square and at least one of the other two submatrices consists entirely of 0's. We will associate to the matrix M a directed graph D(M) and exploit its graphical properties in order to achieve the complete reduction of M into irreducible subgraphs. A directed graph is strongly connected or strong if for any two points, there exists a directed path from each to the other. A strong component is a maximal strong subgraph. It is clear that the points of any directed graph are partitioned into mutually exclusive strong components. Theorem: The irreducible submatrices of M are in a one-to-one correspondence with the sets of points in the strong components of the directed graph D(M). This correspondence enables one to develop

a procedure for reducing a given matrix M by determining the strong components of D(M). The proof is accomplished with the aid of a lemma which coordinates the nonzero terms in the determinant of M with the directed cycles appearing in the strong components of D(M). Clearly the eigenvalues of M are given by the union (including multiplicity) of the sets of eigenvalues of the irreducible submatrices of M. (Received October 9, 1958.)

553-5. Ta Li: On a new class of integral transforms.

In solving the perturbation potential equation, the sink and source solution often leads to the integral equation (1) $\int_\sigma^1 T_n(v/\sigma)Y_n(v)dv/(v^2 - \sigma^2)^{1/2} = f_n(\sigma)$, $n = 0,1,2,3,\ldots$, where $f_n(\sigma)$ is a given function, $T_n(t)$ the Chebyshev polynomial of the first kind and of the nth degree, and $y_n(v)$ is the unknown source strength. The solution of (1) is found to be (2) $Y_n(v) = -(2/\pi)\int_v^1 T_{n-1}(v/\zeta)d[\zeta^n f_n(\zeta)] \cdot (\zeta^{n-1}(\zeta^2 - v^2)^{1/2})^{-1}$ where $T_{-1}(t)$ is interpreted as $T_1(t)$ and $f_n(1)$ is assumed to be zero. (Received October 20, 1958.)

553-6. A. W. Adler and J. J. Kohn: On differentiability properties of Kahler metrics.

The following theorem is proved: Let M be a compact complex analytic manifold which admits a Kahler metric ω_0 of class C^1; then M admits a Kahler metric of class C^∞. The C^∞ Kahler metric is constructed in two stages. First, for each $t \in [-1,1]$ a Kahler metric ω_t is constructed, with the properties that the ω_t vary continuously with t and are C^∞ for $t \neq 0$. Then for each t a projection operator H_t is defined, mapping the space of square integrable forms of type (1,1) onto the solutions of a suitably chosen elliptic operator. It is then proved that $H_t\omega_t$ converges to ω_0, and that for sufficiently small t, ω_t is a Kahler metric. (Received October 20, 1958.)

553-7. D. S. Greenstein: Some new external problems with infinitely many interpolating conditions.

Recently, the author, in collaboration with Bernard Epstein and Jack Minker (Ann. Acad. Sci. Fenn. AI. 250/10), solved the following problems: Let S_σ be the strip $-\sigma < \mathrm{Im} z < \sigma$ and let $L^2(S_\sigma)$ consist of all f(z) regular in S_σ such that $\iint_{S_\sigma} |f(z)|^2 dxdy < \infty$. Given a sequence a_n ($n = 0, \pm 1,\ldots$) of complex numbers. (1) Are there $f(z) \in L^2(S_\sigma)$ such that $f(n\pi) = a_n$? (2) If so, which one has minimum norm? Making further use of Fourier techniques he

developed for the above work, the author is now able to solve the generalized problems in which the interpolating conditions are replaced by $f^{(k)}(n_k) = a_n$ for a fixed non-negative integer k. The author has also made progress on problems of interpolation with minimum norm when infinitely many conditions are given on several consecutive derivatives. (Received October 23, 1958.)

553-8. P. J. Cohen: On the L^1 norm of exponential sums.

It was conjectured by Littlewood that the L^1 norm of the sum of N distinct exponential $e^{in_j x}$, $1 \leq j \leq N$, should be at least as great as K log N for some constant K. However, it was unknown even if the L^1 norm tended to infinity as a function of N. In this paper the author proves the following theorem. Theorem: For some constant K, we have $\|\sum_{j=1}^{N} c_j e^{in_j x}\|_1 \geq K(\log N/\log\log N)^{1/8}$ where $|c_j| \geq 1$. In the special case where $n_j = j$, the exponent 1/8 can be replaced by 1/4. The methods depend upon a combinatorial analysis of the integers n_j together with some general lemmas concerning measures. The method generalizes to the case where $e^{in_j x}$ are replaced by characters on a connected, compact, abelian group, in particular, exponentials in more than one variable. (Received October 24, 1958.)

553-9. Edward Halpern: On the cohomology of a space on which an H-space operates.

The following theorem is proved: Let K be a field of characteristic zero. Let X be an arcwise connected H-space with homotopy-associative and homotopy-commutative multiplication, which operates (on the right-and up to homotopy) on an arcwise connected topological space T. If $f:X \to T$ commutes with the operations of X on T and on itself (by right translations), then $f^*H^*(T,K)$ is a sub-Hopf algebra. Moreover, $H^*(T,K) \cong B \otimes C$, where B and C are subalgebras of $H^*(T,K)$ such that f^* annihilates the positive degree elements of B and is injective on C. This is the analog of a theorem of A. Borel proved where K has characteristic not 2 and assuming $H^*(X,K)$ is an exterior algebra in place of the homotopy-commutativity of X. The theorem applies to Lie groups in two ways: (1) Let T be a connected Lie group, X a closed connected commutative subgroup, and f the inclusion map. (2) Let X be a connected commutative Lie group, T the right coset space modulo a closed subgroup, and f the canonical projection. These give analogous results to theorems of H. Samelson and J. Leray, respectively. (Received October 27, 1958.)

553-10. Samuel Schechter: On the inversion of certain matrices.

Let $H = \{(a_i - b_j)^{-1}\}_1^n$ be a matrix of order n, with a_i, b_j arbitrary distinct complex numbers, and let $H^{-1} = \{c_{ij}\}$. Then it is shown that: (1) $c_{ij} = (a_j - b_i)A_j(b_i)B_i(a_j)$, where the $A_i(x)$, $B_i(x)$ are the fundamental polynomials of the Lagrangian interpolation corresponding to the a_i and b_i respectively; (2) $\sum_{i,j} c_{ij} = \sum_i (a_i - b_i)$. (3) Let $\alpha_i = \sum_j c_{ij}$, $\beta_j = \sum_i c_{ij}$, and let $D_\alpha = [\alpha_1, \ldots, \alpha_n]$, $D_\beta = [\beta_1, \ldots, \beta_n]$ be diagonal matrices. Then $\alpha_i = -A(b_i)/B'(b_i)$, $\beta_j = B(a_j)/A'(a_j)$, and $H^{-1} = D_\alpha H^T D_\beta$, where $A(x) = \prod_i (x - a_i)$, $B(x) = \prod_i (x - b_i)$. The proofs depend largely on Lagrange's interpolation formula. This extends some results of A. R. Collar and R. B. Smith. (Received October 28, 1958.)

553-11. J. B. Roberts: Polynomial identities. I

Let $n_1, n_2, \ldots$ be integers ≥ 2 and define $p_0 = 1$, $p_i = n_1 \ldots n_i$ for $i > 0$. Then if $\gamma_0, \gamma_1, \ldots$ is an arbitrary sequence of complex numbers one can put all integers into 1-1 correspondence with a certain class of finite linear combinations of the γ_i in such a way that $n = a_0 + a_1 p_1 + \ldots + a_k p_k$ is made to correspond to $A_n = a_1\gamma_1 + a_2\gamma_2 + \ldots + a_k\gamma_k$; $0 \leq a_i < n_{i+1}$ for $i \geq 0$. Various identities of the form $\sum_{n=0}^{q-1} f(n) A_n^t = 0$ for $0 \leq t \leq \alpha$ are given where f is a number theoretic function. For example, if $F_q(n) = \prod_{j=1}^{q} f_j(a_{j-1})$ and $\sum_{n=0}^{n_j - 1} f_j(n) n^t = 0$ for $0 \leq t \leq \alpha_j$, $1 \leq j \leq m$, then $\sum_{n=0}^{p_m - 1} F_m(n) A_n^t = 0$ for $0 \leq t < \alpha_1 + \ldots + \alpha_m + m$. Using these results one is able to deduce earlier results of the author as special cases (Amer. Math. Monthly vol. 5 (1957) pp. 317-322; Canad. J. Math. vol. 10 (1958) pp. 191-194). Further one obtains various generalizations of a theorem of Lehmer on sums of like powers (Scripta Math. vol. 13 (1947) pp. 37-41). For example one can split the first p_{m+1} positive integers into $\operatorname{lcm}(n_1, \ldots, n_{m+1})$ classes such that the sum of the A_n^t over the classes is constant for all sufficiently small t. The structure and calculation of these classes is investigated and in the special case where all n_i are equal to $c + 1$ the lower bound $(c!)^2$ to the number of splittings is obtained. (Received October 27, 1958.)

553-12. Avner Friedman: Free boundary problems for parabolic equations.

The following problems are discussed: Stefan type problems (melting of solids), condensation and evaporation of liquid drops, and dissolution of a gas bubble in liquid. The differential equation is the heat equation but all the results remain true for general second order parabolic equations. In all the above problems existence and uniqueness are proved for all future times (in the case of evaporation - only as long as the drop exists; after a finite time the drop evaporates completely). In case of condensation the radius of the drops is shown to grow like $t^{1/2}$ (t = time). The method to prove existence and uniqueness consists in reducing the differential system into a nonlinear integral equation. The last equation is first solved locally and then a step-by-step process is used, deriving first certain a priori estimates. (Received October 30, 1958.)

553-13. Donald Austin: On the entrance-boundary for Markov processes with discrete state space.

Let x(t,w) be a separable, Borel measurable Markov process with right lower semi-continuous sample functions, state space the positive integers, and transition matrix $\{p_{ij}(t)\}$. It is shown that, for almost all sample functions, the following limit exists for all $t_i \geq 0$ (i = 1,2) and all positive integers j, $\lim_{s \downarrow t_1} p_{x(s,w),j}(t_2) = \gamma(t_1,t_2;j,w)$. Furthermore $\sum_j \gamma(t_1,t_2;j,w) = 1$, $\gamma(t_1,t_2+t_3;j,w) = \sum_k \gamma(t_1,t_2;k,w)p_{kj}(t_3)$. If $\alpha(w)$ is a finite, optional random variable then $P[x(\alpha(w)+t,w) = j] = \int_\Omega \gamma(\alpha(w),t;j,w)dP$. (Received October 31, 1958.)

553-14. Karl Stromberg: Probabilities on a compact group.

Let G be an arbitrary compact Hausdorff group and M(G) the dual of C(G), i.e. M(G) = the Banach space of all complex-valued, countably-additive, regular Borel measures of finite variation on G. For $\mu, \lambda \in M(G)$ the measure $\mu^*\lambda$ defined by $\mu^*\lambda(E) = \int_G \mu(Ex^{-1})d\lambda(x)$ is regular and is just that member of M(G) induced by the functional $f \to \int_G \int_G f(xy)d\mu(x)d\lambda(y)$, $f \in C(G)$. With this convolution as multiplication M(G) is a Banach algebra with unit, commutative if G is. The center of M(G) is all μ satisfying the identity $\mu(xEx^{-1}) = \mu(E)$. For $\mu \in M(G)$, the measure $\tilde{\mu}(E) = \int_G \mu(xEx^{-1})dx$ is central and equals μ if μ is central. Let P(G) denote the set of all probability measures on G. With the weak* topology and convolution P(G) is a compact semigroup. For μ in P(G) let $S(\mu)$

be the smallest closed subset of G such that $\mu(S(\mu)) = 1$. If K is the smallest closed subgroup of G containing $S(\mu)$, then the sequence $\{\mu^n: n \geq 1\}$ converges if and only if $S(\mu)$ is contained in no coset of any proper normal closed subgroup of K. If this limit exists, it is the Haar measure on K. If G is finite and $\mu \in P(G)$, then there is a sequence $\{\mu_n: n \geq 1\} \subset P(G)$ containing no units such that $\mu_1 * \ldots * \mu_n \longrightarrow \mu$ if and only if $S(\mu)$ is a union of left cosets of some subgroup H having order > 1. (Received November 6, 1958.)

553-15. Barry Bernstein: A maximum theorem for irrotational subsonic gas flow.

Consider an inviscid gas in steady subsonic adiabatic irrotational flow. Assume that the speed of sound for this gas is a monotonic nondecreasing function of density, as is the case, for example, for an ideal gas. Then the speed may not assume an interior maximum. (Received November 7, 1958.)

553-16. Paul Kelly and E. G. Straus: On the projective centers of convex curves.

Corresponding to a closed convex curve C in the projective plane, there is the group Γ generated by the projective involutions which map C onto itself. Each involution in Γ has a center which is called an interior or exterior projective center of C according as it is interior or exterior to C. The paper is concerned with certain relationships between C and its set of centers. It is shown that if the set of centers is finite, then the number of centers is odd, there is at most one interior center, and the exterior centers are collinear on a line which is not a secant of C. If the centers have a limit point interior to C, then C is a conic. If the centers have a limit point exterior to C then C is differentiable and is the union of two arcs of conics. On the other hand, curves are shown to exist which possess no arc of a conic but which have infinitely many independent interior and exterior projective centers. (Received November 10, 1958.)

553-17. L. A. MacColl: Appell's equations in relativistic particle dynamics.

A particle, subject to a general force, is assumed to move in Euclidean 3-space in accordance with the laws of special relativistic dynamics. A function $S(q^1,\ldots,\dot{q}^1,\ldots,\ddot{q}^1,\ldots)$ is determined such that Appell's equations $\partial S/\partial \ddot{q}^\kappa = Q_\kappa$

give the known differential equations of motion, in terms of general curvilinear coordinates q^{κ}. (Received November 12, 1958.)

553-18. Ward Cheney: Power series for the n-body problem.

Let the i-th body ($1 \leqq i \leqq n$) have mass M_i, and let its h-th Cartesian coordinate ($1 \leqq h \leqq 3$) be expanded in a power series $X^{ih} = \sum_{k=0}^{\infty} X_k^{ih}(t - t_0)^k$. Let initial conditions of position, X_0^{ih}, and velocity, X_1^{ih}, be specified at $t = t_0$. The remaining coefficients in the series may be calculated from the following recurrences, which are derived by techniques similar to Steffensen's. See Math. Rev. 18-263, 19-369. $R_k^{ij} = \sum_{h=1}^{3}\sum_{v=0}^{k}(X_{k-v}^{ih}X_v^{ih} - 2X_{k-v}^{ih}X_v^{jh} + X_{k-v}^{jh}X_v^{jh})$. $S_0^{ij} = (R_0^{ij})^{-3/2}$. $S_k^{ij} = -(2kR_0^{ij})^{-1}[\sum_{v=0}^{k-2} 2(v+1)R_{k-v-1}^{ij}S_{v+1}^{ij} + \sum_{v=0}^{k-1} 3(v+1)S_{k-v-1}^{ij} \cdot R_{v+1}^{ij}]$. $X_{k+1}^{ih} = (k^2 + k)^{-1}\sum_{v=0}^{k-1}\sum_{j=1, j\neq i}^{n} GM_j S_{k-v-1}^{ij}(X_v^{jh} - X_v^{ih})$. These formulas are well-disposed for automatic computation. Convergence rests on Siegel's Vorlesungen über Himmelsmechanik. (Received November 12, 1958).

553-19. R. K. Getoor: A class of integral transforms suggested by probability theory.

Let F be a non-negative bounded Borel measure on the non-negative reals, $r \geqq 0$. We define $\phi_{\mu}(u) = \int_{r \geqq 0} S_{\mu}(ru)dF(r)$ to be the μ-transform of F where $S_{\mu}(r) = 2^{\mu}\Gamma(\mu + 1)r^{-\mu}J_{\mu}(r)$ for $\mu \geqq -1/2$. It is well known that if G is a non-negative bounded Borel measure on E^n which is radially symmetric then the Fourier transform of G is just the $(n-2)/(2)$-transform of $F(r) = \int_{|x| \leqq R} dG(x)$. We study μ-transforms for arbitrary $\mu \geqq -1/2$ obtaining among other results necessary and sufficient conditions that a function ϕ be a μ-transform for all $\mu \geqq -1/2$ and that ϕ be an infinitely divisible μ-transform for all $\mu \geqq -1/2$. (Received November 13, 1958.)

553-20. William Moser: An abstract definition for the Mathieu group of degree twelve.

The Mathieu group M_{12} of degree twelve and order 95040 is generated by the permutations A = (0 1 2 3 4 5 6 7 8 9 10), D = (10 7 2 6)(3 9 4 5) and U = (0 ∞)(1 10)(2 5)(3 7)(4 8)(6 9), (R. D. Carmichael, Introduction to groups of finite order, p. 151). A, D, U and $B = D^2A^2D^2A^6D^2A^2$ = (1 4 5 9 3)(2 8 10 7 6) satisfy the relations $A^{11} = B^5 = D^4 = (AD^2)^3 = (A^{-1}DB)^3 = (UA)^3 = E$, $B^{-1}AB = A^4$, $D^{-1}BD = B^2$, $B = UD^{-1}UD$, $UA^2D^{-1}A^4U = A^{-1}D^2A^3D^2A^4DA^5$. Using the Todd Coxeter enumeration process (Coxeter and Moser, Generators

and relation for discrete groups, Chapter 2) it is shown that this set of relations provides an abstract definition of M_{12}. (Received November 14, 1958.)

553-21. Isidore Fleischer: Best group alphabets.

For transmission over a binary channel where each symbol has a fixed independent probability of being incorrectly received it is for technical reasons desirable to use as the transmitting "alphabet" a subset of the n-fold product of the two element group which forms a subgroup. To correct the most probable errors it should be possible to choose representatives of the quotient group having a small number of ones (this number is called the weight of the element). Following D. Slepian (B. S. T. J. vol. 35 (1956) pp. 203-234) we call a group alphabet best if with any representative all elements of smaller weight are also coset representatives. The present paper uses the fact that, each subgroup being a subspace, it is possible to describe a given alphabet by its complement in the dual space, which is more convenient for the purpose at hand. Identify the dual space with subsets of an n-element set. Theorem: A group alphabet can be used to correct all single errors if and only if the Boolean ring generated by the elements of its complement is the full ring of subsets. This theorem is used to construct simple best alphabets whenever the order of the quotient group does not exceed 2n (the result goes somewhat beyond known cases) as well as for some cases outside this range. This work was done at Bell Telephone Laboratories. (Received November 14, 1958.)

553-22. C. D. Gorman: Brownian motion of rotation.

Let $(x(t), y(t))$, $0 \leqq t < \infty$ be a sample path of the plane Brownian motion process. Let the interval $[0,1]$ be subdivided by the points $k/2^n$, $k = 0,1,\ldots,2^n$, for each $n = 0,1,\ldots$, and consider the polygonal path $(x_n(t), y_n(t))$, $0 \leqq t \leqq 1$ with vertices $(x(k/2^n), y(k/2^n))$, $k = 0,1,\ldots,2^n$. Let S be the unit sphere on which a point P and an oriented great circle thru P have been chosen. Place S on the plane so that P coincides with the origin and the tangent to the great circle with the positive x-axis. Let $R_n(t)$ denote the rotation of S around its center as it rolls without slipping along the polygonal path from 0 to t, $0 \leqq t \leqq 1$. The main theorem states that $\lim_{n \to \infty} R_n(t)$ exists uniformly for t in $[0,1]$ for almost all sample paths $(x(t), y(t))$. (Received November 14, 1958.)

553-23. Philip Dwinger: Retracts in Boolean algebras.

Let A be an α-complete Boolean algebra and let X be a subalgebra of A and let X be α-complete. The α-complete subalgebra of A which is generated by X is denoted by X^*. X is called an α-retract if there exists an α-complete homomorphic mapping of X^* onto X which leaves every element of X fixed. The following is shown. With every X is associated an α-complete ideal I_X of X^* such that; (i) if I is an α-complete ideal of X^* and $I \cap X$ is an α-complete ideal of X then X/I is isomorphic to $X/I \cap X$ if and only if $I \supset I_X$, (ii) X is an α-complete retract if and only if $I_X \cap X = (0)$. A special case of (ii) is Loomis' classical theorem. (Received November 17, 1958.)

553-24. M.- E. Hamstrom: Regular mappings whose inverses are 3-cells.

A proper open mapping f of a metric space X onto a metric space Y is homotopy n-regular provided that for $x \in X$, $e > 0$, there is a $\delta > 0$ such that every mapping of a k-sphere, $k \leq n$, into $S(x,\delta) \cap f^{-1}(y)$, $y \in Y$, is homotopic to 0 in $S(x,e) \cap f^{-1}(y)$ and is completely regular if for each $e > 0$, $y \in Y$, there is a $\delta > 0$ such that $d(y,y') < \delta$, $y' \in Y$, implies the existence of an e-homeomorphism of $f^{-1}(y)$ onto $f^{-1}(y')$. It is proved that if f is a homotopy 2-regular mapping of a metric space X onto a metric space Y where each $f^{-1}(y)$ is homeomorphic to the 3-cell R^3 then f is completely regular. It then follows from results of M.- E. Hamstrom and Eldon Dyer (Completely regular mappings, Fund. Math. vol. 45 (1957) pp. 103-118 and Regular mappings and the space of homeomorphisms on a 2-manifold, Duke Math. J. vol. 25 (1958) pp. 521-532) that if X is complete and Y is finite dimensional then (X,f,Y) is a locally trivial fibre space. Furthermore, if (1) Y is locally compact, separable and contractible or (2) $f | \bigcup \text{bdry } f^{-1}(y)$ corresponds to the projection map of $S^2 \times Y$ onto Y under a homeomorphism h of $\bigcup \text{bdry } f^{-1}(y)$ then X is homeomorphic to $R^3 \times Y$, f corresponding to the projection map, and this homeomorphism extends h if (2) holds. (Received November 17, 1958.)

553-25. Mark Mahowald: Summability in compact abelian groups.

Let G be a compact abelian group and let $\underline{G}$ be the group of characters of G. Formally we have a "Fourier Series", (FS), expansion for a function f in $L_1(G)$ by $\sum_{\underline{G}} f^*(a)a(x)$ where a(x) is a character and $f^*(a) = \int f(x)\bar{a}(x)dx$. This paper studies the pointwise convergence and summability of this series. The definition of summability used is a generalization of Abel summability. The principal results are Theorem A: If G is compact, abelian and not zero-dimensional and if f is in L_1 and is continuous at x, then the FS for f at x is summable to f(x). Theorem B: If G is a compact abelian and zero-dimensional group, then the FS for any f in L_1 converges to f at points of continuity of f. (Received November 17, 1958.)

553-26. S. K. Singh: A note on entire functions. Preliminary report.

In this note by utilizing the properties of proximate order and lower proximate order of entire functions alternative proofs of some theorems of S. M. Shah and R. P. Boas are given. The theorem of Shah (see Math. Student vol. 10, pp. 80-82) concerns the ratio of the rank to the maximum term of an entire function, and the theorem of Boas (see R. P. Boas, Entire functions, New York, 1954, p. 17) states that $\liminf_{r\to\infty}(n(r)/N(r)) \leq \lambda_1$, where n(r) and N(r) have their usual meaning, and $\lambda_1 = \liminf_{r\to\infty}(\log n(r)/\log r)$. Also in this note one proves that for entire functions of mean type $M(r,f') < Kr^{2\rho-1}\mu(r,f)$ where ρ is the order of the function and $\mu(r,f)$ is the maximum term of f(z) for $|z| = r$. Finally by means of an example it is shown that the following result of Valiron namely "$M(r,f') < r^{3/2\rho-1+\epsilon}\mu(r,f)$ for all sufficiently large r except for values of f which can be enclosed in intervals in which the total variation of log r is finite" is best possible in the sense that $3/2\rho$ cannot be replaced by a smaller number (see G. Valiron, Fonctions analytiques, France, 1954, pp. 211-212). (Received November 17, 1958.)

553-27. Joshua Chover: A theorem on integral transforms with an application to prediction theory.

The following theorem is proved: Let $a < b$. Let r(t) be defined on $a - b \leq t \leq b - a$ to be non-negative, continuous, nondecreasing, and concave upwards on $0 \leq t \leq b - a$, and to be even on $a - b \leq t \leq b - a$. Then the range of the integral transform $f(t) = \int_a^b r(t - s)dm(s)$, where $a \leq t \leq b$ and m is a Radon measure on $a \leq s \leq b$, consists precisely of those continuous functions f whose

left-handed derivatives are of bounded variation on $a < t < b$. Method of proof: Conversion to an integral equation of the second kind in a space of measures. Application: For any second-order-stationary, continuous parameter stochastic process with covariance function $r(t)$ having the above properties for $a - b \leqq \tau \leqq b - a$, linear least-squares prediction based on the interval $a \leqq t \leqq b$ can be accomplished by means of a fixed Radon measure on that interval. (Received November 18, 1958.)

553-28. R. G. Laha: On a class of distribution functions where the quotient follows the Cauchy law.

Let x and y be two independently and identically distributed random variables having a common distribution function $F(x)$. Let the quotient $w = x/y$ follow the Cauchy law distributed symmetrically about the origin $w = 0$. Then it is known that $F(x)$ is not necessarily normal [cf. R. G. Laha, Proc. Nat. Acad. Sci. U. S. A. vol. 44 (1958) pp. 222-223; G. P. Steck, Ann. Math. Stat. vol. 29 (1958) pp. 604-606]. The present paper deals with some methods of constructing a class of infinitely many distribution functions $F(x)$ having finite moments up to a certain order K ($K \geqq 1$) where the quotient $w = x/y$ has the Cauchy law. The methods are based on some lemmas which are derived essentially from the theory of analytic functions. (Received November 18, 1958.)

553-29. Aubert Daigneault: The free product of polyadic algebras and applications.

All polyadic algebras (p.a.) considered are assumed to be locally finite and of infinite degree. A B-valued representation f of a p.a. A with domain X is denoted by (f,A,X,B). The free product of a family of p.a. with amalgamation is defined. Existence and unicity are proved. Three interrelated applications are given. I. Theorem 1. Let A_1 and A_2 be p.a. such that $A_1 \subset A_2$ and $p \in A_2 - A_1$. Then there are representations $(f_1,A_2,X,0)$, $i = 1,2$, such that $f_1|A = f_2|A$ and $f_1(p) \neq f_2(p)$. This is an algebraic version of the Beth-Craig-A. Robinson theorem in the theory of definition (Craig, J. Symbolic Logic, 1957, pp. 269-285 and Bibliography given there). II. Let A_1, A_2 and p be as before. p is said to be syntactically semi-definable (sy. s.-d.) with respect to A_1 if there holds a relation of the form $\bigvee_{i=1}^{n}(\forall I)\ (q_i \leqq p) = 1$ where $q_i \in A_1$. p is said to be almost sy. s.-d. if there holds a relation of the form $\bigvee_{i=1}^{n}(\exists I)((\forall J)(q_i \leqq p)) = 1$ where $q_i \in A_1$ and $J = \text{supp. } (p)$. Theorem 2. If p is not almost sy. s.-d. then

there are representations $(f_1, A_2, X, 0)$, $i = 1,2,3,\ldots$ such that $f_i|A_1$ is independent of i and $f_i(p) \neq f_j(p)$ whenever $i \neq j$. This provides an answer to a question raised in the aforementioned paper on p. 280. III. The algebraic counterpart of Craig's interpolation theorem (ibid, p. 267, Theorem 5) is proved. (Received November 24, 1958.)

553-30. Simon Green and W. A. Rutledge: Regular transformations in Euclidean 4-space.

A regular transformation is a projective transformation on an Euclidean 4-space which carries a regular plane into a regular plane, where by a regular plane is meant a plane given by the equation $AX + BY + C = 0$, A, B, C, X, Y being complex numbers. The affine case is an extension of the translation and rotation of J. S. Taylor to the general affine transformation. The general affine transformation is broken down into three elementary transformations and results were obtained similar to those of Taylor, as well as some additional results. In the nonaffine case no projective transformation is regular. The regular plane is transformed into a point. Further investigation revealed that points which are images of parallel planes and do not go to ∞, all lie either on a regular plane or on a nonregular plane. (Received November 19, 1958.)

553-31. A. H. Wallace: Sheets of real analytic varieties.

A real analytic variety V is a closed set in a Euclidean space which is, in a neighbourhood of each of its points, the set of zeros of analytic functions. A subset S of V is a sheet if (1) every pair of points of S can be joined by an analytic curve in S, (2) S is maximal with respect to this property. S is proper if there is an open set U in E such that $U \cap V = U \cap S$. The following are extensions of results already proved for algebraic varieties (Wallace, Algebraic approximation of curves, Canad. J. Math. vol. 10 (1958) pp. 242-278). Theorem 1. Each sheet of maximum dimension is closed. Theorem 2. V is covered by proper sheets. The basic tool is an approximation theorem (slightly weaker than in the algebraic case), namely: If C is a union of analytic curves on V joined end to end, the joins being regular points of V, then an arbitrary neighbourhood of C on V contains an analytic curve with the same end points as C. The essential differences between this situation and that of an algebraic variety V are the absence here of global equations and the fact that the singular locus may not be an analytic variety. (Received November 19, 1958.)

553-32. M. I. Knopp: On certain automorphic functions.

Rayleigh has given the Fourier expansions of the invariants $j(n^{1/2},\tau)$ for the groups $G(n^{1/2})$, $(n = 2,3)$. Here $G(n^{1/2})$ is the group of transformations of the complex plane generated by $\tau' = \tau + n^{1/2}$, $\tau' = -1/\tau$. We derive the functional equations (1) $j(n^{1/2},-1/\tau) = j(n^{1/2},\tau)$, $(n = 2,3)$ directly from these Fourier series. The method is in essence that used by Rademacher in treating $J(\tau)$, the well-known invariant for the full modular group. We next use the Fourier series for $\lambda(\tau)$, derived by Simons, to show directly (2) $\lambda(-1/\tau) = c - \lambda(\tau)$. It is classical that $c = 1$, but this fact does not appear to follow easily from our method. However, it follows immediately from (2) that (3) $\lambda(\tau/(2\tau + 1)) = \lambda(\tau)$. (Received November 20, 1958.)

553-33. R. P. Kanwal: Acoustic streaming past a sphere in a viscous, compressible and heat conducting fluid. Preliminary report.

A spherical oscillator, placed in a uniform flow of speed U of an infinite viscous compressible fluid, is executing radial vibrations. Consequently shear and vorticity waves are generated in the fluid. It is the purpose of this paper to point out the interaction of these waves due to the boundary conditions inherent in the problem. (Received November 20, 1958.)

553-34. Joseph Kist: Invariant positive linear functionals.

The following theorem extends results of Krein and Rutman [Amer. Math. Soc. Translation No. 26, Theorem 3.1]; and Civin and Yood [Pacific J. Math. vol. 6 (1956) Theorem 4.1]. Let E be a normed and partially ordered vector space whose positive cone P is a closed set (it is not assumed that P has interior), and let G be a multiplicative semigroup of continuous positive linear operators acting on E. Suppose that G satisfies the relaxed boundedness condition (i.e., there is a number $K > 0$ such that for each $T \in G_1$, the set of all averages of elements in G, there exists an operator $V \in G_1$ such that $\|V\| \leq K$, $\|VT\| \leq K$), and that G is either (i) a left solvable semigroup for which $G^{(i)}$, $i = 0,\ldots,n$ ($G^{(0)} = G, G^{(i+1)}$ is a left commutator subsemigroup of $G^{(i)}$, and $G^{(n)}$ is commutative) satisfies the r.b.c., or (ii) a group, every finite subset of which is contained in a compact subgroup of G. Then, the existence of an element $x \notin -P$, and a positive number σ such that $Tx \geq \sigma x$ for all $T \in G$ implies that E admits a nonzero continuous positive linear functional which is invariant under G. (Received November 20, 1958.)

553-35. G. H. Meisters: Almost periodic motions in uniform spaces.

This paper generalizes Hans Tornehave's theory of almost periodic motions in metric spaces [Dan. Mat. Fys. Medd. vol. 28, no. 13 (1954)] to a theory of almost periodic motions in uniform spaces. Let $(X,\mathcal{U})$ be a uniform space, and let f(t) be a function defined and continuous for all real t with values in X. Then f(t) is called an almost periodic motion in X iff for each $U \in \mathcal{U}$ there exists a relatively dense set of real numbers τ such that $(f(t+\tau), f(t)) \in U$ for all real t. All the main theorems of Tornehave's theory go through with little change, and in particular if $(X,\mathcal{U})$ is complete and continuously locally arcwise connected, the Approximation Theorem holds. Here $(X,\mathcal{U})$ is called complete iff every closed and totally bounded subset is compact, and $(X,\mathcal{U})$ is called continuously locally arcwise connected iff for each compact set C of X there exists a $U_C \in \mathcal{U}$ and a function $\phi(x;t;y)$, defined and continuous for $0 \leqq t \leqq 1$ and $(x,y) \in U_C \cap (C \times C)$, with values in X such that $\phi(x;0;y) = x$, $\phi(x;1;y) = y$, and $\phi(x;t;x) = x$. (Received November 20, 1958.)

553-36. Anthony Trampus: Differentiability and analyticity of functions in linear algebras.

Let $\mathcal{A}$ be a linear algebra of dimension n with unit element over the real or complex field, and f a function whose domain and range are subsets of $\mathcal{A}$. Let f possess a Fréchet differential df(a,h) at a point $a \in \mathcal{A}$. We define $f^1(a) = df(a,)$ to be the derivative of f at a. Now $f^1(a) \in \mathcal{T}_n$, the algebra of all linear transformations in $\mathcal{A}$, and the matrix representing $f^1(a)$ is the Jacobian matrix of the component functions $\phi_1,\phi_2,\ldots,\phi_n$ of f with respect to the components $\xi_1,\xi_2,\ldots,\xi_n$ of a point in $\mathcal{A}$. The function f is analytic at a if $f^1(a) \in \mathcal{T}$, the enveloping algebra of left and right multiplications in $\mathcal{A}$. These definitions are analogous to those of J. A. Ward (Duke Math. J. vol. 7(1940) pp. 233-248) who employed only matrix representations in developing a function theory in linear algebras. This paper extends Ward's results and at the same time sheds new light on the connections among the works of Ward, Ringleb (Rend. Circ. Mat. Palermo vol. 57 (1933) pp. 311-340) and R. W. Wagner (Duke Math. J. vol. 9 (1942) pp. 677-691). (Received November 21, 1958.)

553-37. A. A. Goldstein: Taylor series for the regularized N-body problem.

By application of an idea due to Steffensen (Mat.-Fys. Medd. Danske Vid. Selsk. vol. 30, no. 18 (1956) and vol. 31, no. 3 (1957)) recursion formulae are given for the coefficients of the Taylor series of the coordinates x_{ih} defined by the differential equations $x''_{ih}U^2 + x'_{ih}UU' = (1/m_i)(\partial U/\partial x_{ih})$ i = 1,2,...,n,h = 1,2,3. Here primes indicate derivatives with respect to Sundman's uniformizing variable u where du/dt = U. Here and above U is the potential energy of the system. (Received November 24, 1958.)

553-38. Johann Sonner: On limits of module-systems.

This paper is concerned with the construction of limits of generalized module-systems and a cofinality-theorem (see Griffiths, Proc. Amer. Math. Soc. vol. 9, no. 1 (1958). A pair (f,g), where f is a surjective map of E into M and g a map of M into $\mathfrak{P}(E^2)$ defines on E, M a structure of an R-module-system, if it possesses the following properties: For all $\alpha,\beta \in M$, (SM_I) $f^{-1}(\alpha)$ is an R-module, (SM_{II}) $g(\alpha,\beta)$ is a submodule of $f^{-1}(\alpha) \times f^{-1}(\beta)$, $(SM_{III})g(\alpha,\alpha)$ is a part of the diagonal of $f(\alpha)\times f(\alpha)$. For every filter $\mathfrak{f}$ on M, an R-module $\lim_{\mathfrak{f}}(f,g)$ can be constructed by means of g-admissible local sections. A cofinality property (CF) is introduced and the following theorems are proved: Theorem 1. Let E,M be an R-module-system and $\mathfrak{f}, \mathfrak{f}'$ filters on M with $\mathfrak{f}'$ finer than $\mathfrak{f}$. If $(f,g,\mathfrak{f},\mathfrak{f})$ satisfies (CF) then $\lim_{\mathfrak{f}}(f,g) = \lim_{\mathfrak{f}'}(f,g)$. Theorem 2. Let E,M be an R-module-system, N a part of M and $\mathfrak{f}$ a filter on N. If $\mathfrak{f}$ is the filter on M generated by $\mathfrak{f}$, then $\lim_{\mathfrak{f}'}(f,g) = \lim_{\mathfrak{f}}(f_N,g_N)$. Theorem 3. Let (f,g), f',g') be structures of R-module-systems on E,M, $\mathfrak{f}, \mathfrak{f}'$ filters on M and N a part of M. Suppose that the induced structures and filters on N exist and coincide, and that $(f,g,\mathfrak{f},\mathfrak{f}_N)$, $(f',g',\mathfrak{f}',\mathfrak{f}_N)$ satisfy (CF). Then $\lim_{\mathfrak{f}}(f,g) = \lim_{\mathfrak{f}'}(f',g')$. (Received November 24, 1958.)

553-39. N. L. Alling: On certain totally ordered non-abelian groups of rank 2.

Let G be a totally ordered (multiplicative) group of rank 2. Let 1 and K be its only proper isolated subgroups. If the center of G does not contain K and if G is complete, in the order topology, then there exists a subgroup H of G such that G can be written as the subdirect product of H and K. The author has shown, in an earlier paper, that order preserving homomorphisms a and b from G and

K into the additive group of real numbers, having as their kernels K and 1 respectively, exists. G can be mapped by an order preserving isomorphism onto a (G) X b(K) where a(G) X b(K) is lexicographically ordered and is rendered a group by the following: $(x,y)(x',y') = (x + x', ye^{tx'} + y')$. Finally, the real number t is uniquely determined by G. (Received November 25, 1958.)

553-40. F. H. Brownell: Explicit perturbation formulae and convergence theorems.

In complex Hilbert space X let H_0 be a self-adjoint operator with λ_0 a not necessarily isolated point of the H_0 spectrum, which is in the point spectrum with positive finite dimensional eigenspace $M_0 = E_0(\{\lambda_0\})X$, E_0 being the spectral measure of H_0. Define the pseudo-inverse $B_0 = \int_{\lambda \neq \lambda_0} (\lambda - \lambda_0)^{-1} dE_0(\lambda)$, self-adjoint but not necessarily bounded. For complex s we consider the perturbation problem for the perturbed operator $H_0 + sT$ of finding $\dim M_0$ many, linearly independent s power series $u + \sum_{p=1} s^p v_p$, $u \in M_0$ and $v_p \in X$, and associated complex power series $\lambda_0 + \sum_{p=1} \gamma_p s^p$ satisfying in the formal s power series sense $(H_0 + sT)(u + \sum_{p=1} s^p v_p) = (\lambda_0 + \sum_{p=1} \gamma_p s^p)(u + \sum_{p=1} s^p v_p)$. Assuming for T only that all iterative products K of B_0 and T, with all T factors separated by positive powers of B_0, contain M_0 in their domain, we have the results abbreviated roughly as follows. (I) We obtain a general inductive construction of all solutions $\{\gamma_p\}$ and $\{v_p\}$ of this problem, subject to a Hermitian condition at each pth stage, for all possible splitting cases of the $\{\gamma_p\}$. (II) For two low splitting cases (the methods appear applicable to any such) this Hermitian condition holds for a class of T properly including the symmetric operators and explicit, noninductive formulae for γ_p and v_p are given. (III) Assuming in addition only a boundedness condition on the above products K, sharp radii of convergence are found for these $\{\gamma_p\}$ and $\{v_p\}$ series in II). (Received November 17, 1958.)

553-41. A. B. Simon: On the maximality of vanishing algebras.

It has been shown by John Wermer that if G is a discrete subgroup of the reals or if G is the real line, then the set of all L^1 functions which vanish on the negative elements is a maximal proper closed subalgebra of $L^1(G)$. In this note we prove the following converse: Let G be a locally compact abelian group, let S be a Haar measurable subsemigroup of G, and let L_S be the set of all L^1 functions on G whose supports lie in S. If L_S is a maximal proper closed

subalgebra of $L^1(G)$, then G is (continuously) isomorphic with either a discrete subgroup of the reals or the whole real line. (Received November 20, 1958.)

553-42. G. T. Cargo: Boundary behavior of Blaschke products.

Let $0 < |a_n| < 1$ $(n = 1,2,\ldots)$ and $\sum(1 - |a_n|) < \infty$. Then the Blaschke product $B(z;\{a_n\})$ with the zeros a_n converges in $|z| < 1$. Theorem 1. Let $0 < d_n < 1$ $(n = 1,2,\ldots)$, $\sum_{n=1}^{\infty} d_n < \infty$, and $\sum_{n=2}^{\infty} d_n \cdot \log n = \infty$. Then there exists a sequence $\{a_n\}$, where $|a_n| = 1 - d_n$, with property (P): Corresponding to each countable point set $E = \{e^{i\theta_n}\}$ on the unit circle C, there exists a subsequence $\{a_{n_k}\}$ of $\{a_n\}$ such that $B(z;\{a_{n_k}\})$ fails to have a radial limit at each point of E. This extends a result of Frostman. The theorem still holds if the class of countable point sets E on C is replaced by a certain class of thicker sets on C. Theorem 2. A necessary and sufficient condition for a Blaschke product to satisfy condition (P) is that $\sum_{n=1}^{\infty}(1 - |a_n|)/|e^{i\theta} - a_n| = \infty$, for all θ. Theorem 3. If a Blaschke product $B(z;\{a_n\})$ has infinite radial variation at $e^{i\theta}$, that is, if $\int_0^1 |B'(re^{i\theta};\{a_n\})|dr = \infty$, then some subproduct $B(z;\{a_{n_k}\})$ fails to have a radial limit at $e^{i\theta}$. (Received November 26, 1958.)

553-43. L. W. Green: On a problem of Blaschke.

Call the distance along a directed geodesic to the first point focal to the geodesic orthogonal at the initial point, the focal distance for the initial line element. Theorem: Suppose, on a C^2 Riemannian manifold homeomorphic to the two-sphere, the focal distance is independent of the initial line element. Then the Gaussian curvature is constant. The hypothesis implies that the distance between (first) conjugate points along geodesics is also constant, and the truth of the theorem with this weaker condition would answer affirmatively a long-standing conjecture of Blaschke. As it is, this establishes for the Riemannian case a theorem proved by Busemann only for dimensions greater than two in general G-spaces. The proof is by analysis of the Jacobi differential equation. (Received November 26, 1958.)

553-44. M. D. Marcus and B. N. Moyls: Linear transformations on algebras of matrices, I.

Let M_n denote the set of $n \times n$ matrices over the complex numbers; and let U_n, H_n, R_k denote respectively the unimodular group, the set of Hermitian matrices and the set of matrices of rank k in M_n. Let ev(A) be the set of n

eigenvalues of $A \in M_n$ counting multiplicities. We determine the structure of any linear T mapping M_n into M_n having one or more of the following properties: (a) $T(R_k) \subseteq R_k$, $k = 1,\ldots,n$, (b) $T(U_n) \subseteq U_n$, (c) $\det T(A) = \det A$ for all $A \in H_n$, (d) $ev(T(A)) = ev(A)$ for all $A \in H_n$. We prove first that if T has property (a) then there exist nonsingular U and V in M_n such that either $T(A) = UAV$ for all $A \in M_n$ or $T(A) = UA'V$ for all $A \in M_n$. We then show that any T satisfying (b), (c), or (d) must in turn satisfy (a) and determine the additional restrictions on U and V required in these cases. For $n = 2$, (c) reduces to the determination of the Lorentz group. As a corollary we have that any linear transformation on $n \times n$ matrices to $n \times n$ matrices that preserves the eigenvalues must be a similarity transformation $A \longrightarrow SAS^{-1}$ (modulo taking the transpose). This work was supported by a National Science Foundation Research Grant and is in press with Canad. J. Math. (Received November 26, 1958.)

553-45. Y. S. Chow: Pointwise convergence of martingales with a directed index set.

Let (Ω, B, u) be a σ-finite measure space and (x_h, B_h, H) a martingale, where H is a directed set and B_h is a σ-algebra of measurable sets such that $B_{h'} \subseteq B_h$ for $h' < h$ and x_h B_h-measurable. If, for every set $A \in B_h$ of finite measure, $\sup \int_A |x_h| du < \infty$, $h \in H$, then $\lim x_h$ exists a.e. and $\int_A \lim |x_h| du < \infty$, provided that a certain Vitali-type condition is satisfied. Krickeberg (Trans. Amer. Math. Soc. vol. 83 (1956) p. 337) has obtained a similar result under the condition that there exists a countable cofinal subset of H. An application of this theorem is a new proof of the Lebesgue differentiation theorem of an interval function of founded variation (Saks, Theory of the integral, p. 115). By using a semimartingale analogue of a lemma in Saks (ibid, p. 135), a convergence theorem is obtained: Let $(x_n, B_n, n \geq 1)$ be a semimartingale where B_n is generated by disjoints atoms I_i^n, $i = 1,2,\ldots$, of finite positive measure and $\inf [m(I_j^{n+1})/m(I_j^n)] > 0$, $i,j,n \geq 1$. Then, $\lim x_n$ exists and is finite a.e. where $\limsup x_n < \infty$. (Received November 28, 1958.)

553-46. Murray Gerstenhaber: Nilalgebras and linear varieties of nilpotent matrices, IV.

All linear varieties of nilpotent matrices of maximal dimension subject to certain natural conditions are determined and shown to be nilpotent associative algebras: If A, B are matrices over a field K, define $A \succ B$ if for some

X, B is a specialization of $X^{-1}AX$. If a nilpotent $n \times n$ matrix A is similar to a direct sum of irreducible matrices of dimensions $k_1,\ldots,k_r$, where (k_i) is necessarily a partition of n, define $t^*(A)$ to be the dual partition of n. Theorem: Let V be a linear variety of nilpotent $n \times n$ matrices and M be a matrix such that $M \to A$ for all A in V. Set $t^*(M) = (b_1,\ldots,b_s)$. Then (i) $\dim V \leq (n^2 - \sum b_i^2)/2$ (ii) if $\dim V = (n^2 - \sum b_i^2)/2$ then there exists a permutation $(b_1',\ldots,b_s')$ of $(b_1,\ldots,b_s)$ such that V is similar to the algebra of all matrices A for which $A_{ij} = 0$ if $j < i$ or if $\sum_{h=1}^{k} b_i' < i,j \leq \sum_{h=1}^{k+1} b_i$, i.e., A is zero below the diagonal and in a set of disjoint diagonal blocks which cover the diagonal and have dimensions $b_i' \times b_i'$, $i = 1,\ldots,s$. (Received November 28, 1958.)

553-47. J. H. Hodges: Some exponential sums for matrices over a finite field. Preliminary report.

Let $GF(q)$, $q = p^n$ denote the finite field of order q. For $\alpha \in GF(q)$, let $e(\alpha) = \exp(2\pi i t(\alpha)/p)$, where $t(\alpha) = \alpha + \alpha^p + \ldots + \alpha^{p^{n-1}}$. For q odd, let $S^+(t,k) = \sum e(|A_1| + \ldots + |A_k|)$, where the summation is over all sets $A_1,\ldots,A_k$ of k symmetric matrices of order t over $GF(q)$. The value of this sum is found in terms of known Gauss sums and the number $N(t,t,\lambda)$ of symmetric matrices of order t, rank t and invariant λ over $GF(q)$. [$N(t,t,\lambda)$ is given by L. Carlitz in Representations by quadratic forms in a finite field, Duke Math. J. vol. 21 (1954) pp. 127-128.] Corresponding sums are considered with simpler results for arbitrary matrices of order t and for skew matrices of order t over $GF(q)$, and for Hermitian matrices of order t over $GF(q^2)$. A number of other exponential sums of various types are also evaluated. (Received November 28, 1958.)

553-48. G. L. Krabbe: A space of multipliers of Fourier integrals.

Suppose $1 < p \leq 2$. Let $\mathfrak{M}_p$ be the set of all factor functions for Fourier transforms of type (L_p,L_p), the terminology being the one found in [Hille and Phillips, Amer. Math. Soc. Colloquium Publication, vol. 31, 21.2]. THEOREM: If a is a bounded Radon measure on $(-\infty,\infty)$, then $a \in \mathfrak{M}_p$. If $1 < p < \infty$, then this theorem is valid, provided $\mathfrak{M}_p$ denotes a slightly wider class than Mihlin's Multipliers of Fourier integrals, [Dokl. Akad. Nauk SSSR vol. 109 (1956) pp. 701-703]. There exists a number k having the following property. Suppose that a is an integrable function which is of bounded variation on $(-\infty,\infty)$ and whose Fourier transform A is integrable: then $\|A\|_1 \leq k \cdot V(a)$, where $\|\cdot\|_1$ denotes the norm in $L_1(-\infty,\infty)$ and $V(a)$ = the total variation of a on $(-\infty,\infty)$. These results

are obtained by straightforward generalization of a theorem by Stečkin, [Dokl. Akad. Nauk SSSR vol. 71 (1950) pp. 237-240]. (Received November 28, 1958.)

553-49. L. R. McCulloh: Integral bases in Kummer extensions of algebraic number fields. Preliminary report.

Let F' be a separable extension of F where F is the quotient field of a Dedekind ring. What are necessary and sufficient conditions for F' to have an integral basis over F? Mann has given the answer in case F' is a quadratic extension of F (Proc. Amer. Math. Soc. vol. 9 (1958) pp. 167-172). This paper gives the answer if F is an algebraic number field containing the ℓ-th roots of unity (ℓ an odd prime) and $F' = F(\mu^{1/\ell})$, where $\mu \in F$. The condition given is that the square root of the discriminant F' over F should be a principal ideal in F. This condition is decidable in F without reference to the arithmetic of F'. Necessity is well known. (See above reference.) Sufficiency is shown by a construction involving a suitable modification of the field basis $1, \mu^{1/\ell}, \ldots, (\mu^{1/\ell})^{\ell-1}$. (Received November 28, 1958.)

553-50. H. B. Mann: The algebra of a linear hypothesis.

We consider the algebra A generated by the matrices of the normal equations arising in the tests of a sequence of linear hypotheses. The decomposition of the sums of squares for the analysis of variance corresponds to a decomposition of A into a direct sum of left ideals. The principal components of A are obtained in the case that A is generated by two idempotent generators G_1 and G_2. In this case every characteristic root λ of $G_1G_2G_1$ corresponds to exactly one principal component except the root 0 which corresponds to at most two principal components. The principal components corresponding to 0 and 1 are 1×1 real complete matrix algebras, those corresponding to the other roots are 2×2 real complete matrix algebras. If the root λ corresponds to the principal idempotent f then the efficiency of the design with respect to the row vectors of f is $1 - \lambda$. The principal decomposition of the algebra of s dimensional cubic lattice designs is explicitly obtained. (Received November 28, 1958.)

553-51. H. D. Mills: Noncooperative games and nonlinear programs.

A finite n-person game in normalized form, characterized by a vector of multilinear forms $(A_1(x), \ldots, A_n(x))$, where $x = (x_1, \ldots, x_n)$, $x_i = (x_{i1}, \ldots, x_{im_i})$, $x_{i1} + \ldots + x_{im_i} = 1$, $x_{ij} \geq 0$, has a noncooperative solution x^0 if and only if

(1) x^0 and $\alpha^0 = (A_1(x^0),\ldots,A_n(x^0))$ solve the nonlinear program: max $A_1(x) + \ldots + A_n(x) - \alpha_1 - \ldots - \alpha_n$ when $\partial A_i(x)/\partial x_{ij} \leq \alpha_i$, $x_{il} + \ldots + x_{im_i} = 1$, $x_{ij} \geq 0$, or (2) there exists α^0 such that $A_1(x^0) + \ldots + A_n(x^0) = \alpha_1^0 + \ldots + \alpha_n^0, \partial A_i(x^0)/\partial x_{ij} \leq \alpha_i^0$, $x_{il}^0 + \ldots + x_{im_i}^0 = 1$, $x_{ij}^0 \geq 0$. Thus, in addition to existence, noncooperative solutions are guaranteed which are algebraic in the coefficients of the characterizing multilinear forms. (Received November 28, 1958.)

553-52. T. J. Rivlin: On the maximum modulus of polynomials.

Let $p = p(z)$ denote a complex polynomial of degree n. Let $M(p,r) = [\max|p(z)|, |z| = r]$. $p \in P \leftrightarrow M(p,1) = 1$. $p \in Q \leftrightarrow p \in P$ and $p(z) \neq 0$ for $|z| < 1$. The following three results are known: A. If $p \in P$ then $M(p,R) \leq R^n$, $R \geq 1$. B. If $p \in Q$ then $M(p,R) \leq (1 + R^n)/2$, $R \geq 1$. C. If $p \in P$ then $M(p,r) \geq r^n$, $r \leq 1$. For A and B see Ankeny and Rivlin (Pacific J. Math. vol. 5 (1955) pp. 849-852) and references there. For C see Varga J. Soc. Indust. Appl. Math. vol. 5 (1957) p. 44). The following result complementary to C (as B is to A) is proved: (1) If $p \in Q$ then $M(p,r) \geq ((1 + r)/2)^n$, $r \leq 1$, with equality only for $p = ((a + bz)/2)^n$, $|a| = |b| = 1$. Combining (1) with B yields: (2) If $p \in P$ has all its zeros on $|z| = 1$ then $((1 + r)/2)^n \leq M(p,r) \leq (1 + r^n)/2$, $r \geq 0$. (Received November 28, 1958.)

553-53. R. F. Rinehart: Skew matrices as square roots.

Skew hermitian matrices are characterized as the normal square roots of negative definite, or semidefinite, hermitian matrices; real skew matrices as the normal real square roots of negative definite, or semidefinite, real symmetric matrices whose nonzero eigenvalues have even multiplicities. It is shown that, in either case, all (all real) square roots of the same rank as the given hermitian (real,symmetric) matrix are obtained by similarity transformations of the skew hermitian (real,skew) square roots. (Received November 28, 1958.)

553-54. Peter Scherk: On regular bilinear forms.

Let $\{x,y\}$ be a regular bilinear form in a finite dimensional vector space E over a field of characteristic $\neq 2$. It determines uniquely a linear transformation t of E such that (*) $\{tx,y\} = \{y,x\}$ for all x,y in E. This transformation is orthogonal with respect to the symmetric part $(\{x,y\} + \{y,x\})/2$ of our form and symplectic with respect to its skew-symmetric part given conversely any

symmetric bilinear form (x,y) in E and an orthogonality t with respect to it. All the regular bilinear forms $\{x,y\}$ with the symmetric part (x,y) are determined which satisfy (*). (Received November 28, 1958.)

553-55. Ernst Snapper: Polynomials associated with divisors.

Let X be an irreducible, normal, projective variety, defined over an arbitrary groundfield. It is proved (These Notices Amer. Math. Soc. vol. 5, August, 1958, p. 477) that, if $D_1,\ldots,D_n$ are divisors of X which are locally linearly equivalent to zero and F an algebraic coherent sheaf of X, the Euler-Poincaré characteristic $\chi(X,F(x_1D_1 + \ldots + x_nD_n))$ is a polynomial with rational coefficients for all rational integers $x_1,\ldots,x_n$. Put $C(i) = x(x + 1)\ldots(x + i - 1)/i!$ and $[i_1,\ldots,i_n] = \prod_{j=1}^{s} C(i_j)$ and expand $\chi(X,F(x_1D_1 + \ldots + x_nD_n)) = \sum a(i_1,\ldots,i_n)[i_1,\ldots,i_n]$, where $i_1,\ldots,i_n \geqq 0$. Observe that C(i) is not the usual "binomial coefficient". Nevertheless, each $a(i_1,\ldots,i_n)$ is a rational integer. Precisely, $a(i_1,\ldots,i_n)$ is the proper definition of "the virtual arithmetic genus with values in F of the set of divisors $D_1,\ldots,D_n$ taken with, respectively, the multiplicities $i_1,\ldots,i_n$." As immediate consequences we obtain relations between virtual arithmetic genera of sums and intersections of divisors, between virtual arithmetic genera, virtual degrees and intersection numbers of divisors, and formulas which deal with intersection numbers of divisors with canonical divisors. Finally, various forms of the Riemann-Roch theorem are derived painlessly for higher dimensional varieties. (Received November 28, 1958.)

553-56. J. D. Swift, Marshall Hall, Jr. and R. Killgrove: On planes of order nine.

A search was made for all planes of order nine which, in some ternary ring, have an elementary Abelian group for addition. It was found that this property is possessed by the four distinct planes of order nine already known, and by no further planes. It is however true that, excepting the Desarguesian plane, the elementary Abelian addition arises in each plane in more than one ternary ring, and these are not all isotopic to each other. This search was carried out in part by hand and in part on the SWAC computer. (Received November 28, 1958.)

553-57. T. E. Stern and Bernard Friedland: Application of modular sequential circuits to single error-correcting p-nary codes.

Systematic methods for the construction of close-packed multi-level single error-correcting codes, coders and decoders are given. Coding and decoding are achieved through the use of linear modular sequential circuits. The synthesis of an appropriate decoding filter is seen to be based upon the selection of a matrix over the modular field GF(p), whose powers display a periodicity with maximal length period. The properties of these matrices are shown to depend upon the properties of the elements of a Galois field, and several theorems from Galois theory are used in the selection of the decoding filter. (Received November 28, 1958.)

553-58. Yousef Alavi and C. P. Wells: Expansions of parabolic wave functions.

The main result of this paper is an expansion of parabolic wave functions in terms of spherical wave functions. The coefficients of expansion are determined in terms of Pasternack's functions (Phil. Mag., 7th series, vol. 28 (1939) pp. 209-226). With the orthogonality condition known for these functions (Bateman, Proc. Nat. Acad. Sci. U. S. A. vol. 28 (1942) pp. 371-375), the series can be inverted giving spherical wave functions as integrals of parabolic wave functions. Special cases include an expansion due to Hochstadt (Proc. Amer. Math. Soc. vol. 8 (1957) pp. 489-491) and the limiting case where the wave functions become potential functions. (Received December 1, 1958.)

553-59. G. B. Dantzig: A decomposition principle for linear programs.

A vector P interconnecting two parts of a program is viewed as drawn from convexes C and C' defining the parts. P is represented as a convex combination of a finite set of points of C and equated to a similar representation in C'. When C and C' are defined by sets of linear inequalities, the selected points correspond to extreme points and certain homogeneous solutions of closely associated convexes. Starting with some admissible m-component vector $P = P^0$, the total points used in the two representations can be reduced to m + 2. This forms a basis whose Lagrange (Simplex) multipliers π relative to a form being extremized are used to test optimality or for generating a better inter-connecting vector by solving two independent linear programs for points in C and C' which minimize two linear forms dependent on π. Computationally

if P is an m-component vector and C is defined by m + k equations in n non-negative variables and C' by m + k' equations in n' non-negative variables, then each major simplex cycle consists in solving two k × n and k' × n' auxiliary linear programs after m(m + n + n') multiplications to set up π and adjust the solution. The iterative procedure is finite. (Received December 1, 1958.)

553-60. Abolghassem Ghaffari: The behavior of Rayleigh's equation at infinity.

The purpose of this paper is to discuss the critical points of Rayleigh's equation (1) $d^2x/dt^2 + \mu[-dx/dt + 1/3(dx/dt)^3] + x = 0$, $\mu > 0$, at infinity and analyze its full phase-portrait. The existence and uniqueness of a stable limit-cycle of (1) is proved by J. J. Stoker (Nonlinear vibrations). Following S. Lefschetz (Differential equations: Geometric theory) one can obtain the following results: The critical points on the surface of the unit-sphere are defined by $(0,0,\pm 1)$, which correspond to the critical point of (1) at the origin (unstable focus). The arcs of the circle $z = 0$ (equator) are paths of (1). The only critical points of (1) on the equator are defined by the intersection of the lines $x = 0$, $y = 0$ with the unit-sphere, which are located at the ends of diameters parallel to the x and y axes, and the paths of (1) outside the limit-cycle tend toward the limit-cycle. (Received December 1, 1958.)

552-61. Casper Goffman and Daniel Waterman: On upper and lower limits in measure.

D. E. Menchoff (On convergence in measure of trigonometric series, Amer. Math. Soc. Translation, no. 105) introduced the notion of upper and lower limits in measure of extended real-valued measurable functions. A definition is given here which is easily shown to be equivalent to that of Menchoff and which greatly simplifies the derivation of the main properties. Only those measurable functions whose values are in $[-1, 1]$ need be considered. Let $\mathfrak{M}$ be the set of equivalence classes of such functions. $f \geq g$ if $f(x) \geq g(x)$ almost everywhere. $\mathfrak{M}$ is a complete lattice. If $\{f_n\}$ is a sequence in $\mathfrak{M}$, u is an upper function relative to $\{f_n\}$ if $\lim_{n\to\infty} m(S_n) = 0$, where S_n is the set for which $f_n(x) > u(x)$. The infimum U of the set of upper functions is the upper limit in measure of $\{f_n\}$. The lower limit is defined analogously. Another definition, which further illuminates the nature of these limits, may be given. A sequence $\{f_n\}$ is equivalent to $\{g_n\}$ if $\{f_n - g_n\}$ converges in measure to zero. Let $\mathfrak{F}$ be

the set of all sequences equivalent to $\{f_n\}$. Then $U = \inf[\lim\sup_{n\to\infty} g_n(x)$: $\{g_n\} \in \mathcal{F}]$ and $L = \sup[\lim\inf_{n\to\infty} g_n(x)$: $\{g_n\} \in \mathcal{F}]$. (Received December 1, 1958.)

553-62. G. M. Helmberg: A theorem on equidistribution in compact groups. II.

The results of a previous paper (Bull. Amer. Math. Soc. Abstract 63-4-583) are generalized. The main results are: Theorem 1: Let $\{H_k: k = 1,2,\ldots,n\}$ be a system of subgroups of a compact group G, satisfying the following conditions: (a) for every $k = 1,2,\ldots,n$ there is given a sequence $\{h_{kj}: j = 1,2,\ldots\}$ that is equidistributed in H_k; (b) for every nontrivial irreducible representation $R^{(\lambda)}$ of G there is at least one index $\bar{k}$ such that $R^{(\lambda)}$, restricted to $H_{\bar{k}}$, does not contain the trivial representation of $H_{\bar{k}}$. Let all elements of the form $h_{1j_1}h_{2j_2}\ldots h_{nj_n}$ $(j_k = 1,2,\ldots)$ be arranged to a sequence $\{g_j: j = 1,2,\ldots\}$ in such a way that for all $i = 1,2,\ldots$ the elements g_j $(1 \leqq j \leqq i^n)$ are exactly all elements of the form $h_{1j_1}h_{2j_2}\ldots h_{nj_n}$ $(1 \leqq j_k \leqq i)$. The sequence $\{g_j: j = 1,2,\ldots\}$ is then equidistributed in G. Theorem 2. Let $\{H_k: k = 1,2,\ldots,n\}$ be a system of separable subgroups of the compact group G, satisfying condition (b) of Theorem 1. Then there is $G = H_1H_2\ldots H_n$. (Received December 1, 1958.)

553-63. J. A. Hummel: Extremal problems for starlike functions.

In a previous paper (Proc. Amer. Math. Soc. vol. 9 (1958) pp. 82-87) the writer introduced a variational method for starlike functions and showed that the maximizing or minimizing functions for extremal problems on the coefficients gave radial slit mappings. The present result shows that the same conclusion holds for a broad class of extremal problems and gives a method of determining an upper bound for the number of slits. A typical simple application is that the set of all possible values of $f(z_1)/f(z_2)$ for f in the class of starlike functions and fixed z_1, z_2 is the region bounded by the set of all possible values of this ratio when f is a single slit (Koebe) function. (Received December 1, 1958.)

553-64. D. G. Johnson: A structure theory for a class of lattice-ordered rings. I. Preliminary report.

An f-ring is a ring A which is partially ordered, and such that (i) (A, +) is an ℓ-group; (ii) if $a, b \geq 0$, then $ab \geq 0$; (iii) if $a \wedge b = 0$ and $x \geq 0$, then $xa \wedge b$

$= ax \wedge b = 0$; see Birkhoff and Pierce (An. Acad. Brasil Ci. 28 (1956), 41-69). An ideal I of A is an $\mathscr{l}$-ideal if $a \in I$ and $|x| \leqq |a|$ imply $x \in I$; one sided $\mathscr{l}$-ideals are defined similarly. A right $\mathscr{l}$-ideal I is modular if there is an $e \in A$ such that $x - ex \in I$ for all $x \in A$. An element $a \in A$ is said to be right $\mathscr{l}$-quasi-regular ($\mathscr{l}$-QR) if the smallest right $\mathscr{l}$-ideal containing $\{x - ax: x \in A\}$ is all of A. A right $\mathscr{l}$-ideal I is $\mathscr{l}$-QR if every $a \in I$ is a right $\mathscr{l}$-QR element. An $\mathscr{l}$-ideal P of A is $\mathscr{l}$-primitive iff there is a maximal modular right $\mathscr{l}$-ideal M of A such that P is the largest $\mathscr{l}$-ideal of A contained in M. The J-radical J(A) of A is the intersection of all maximal modular right $\mathscr{l}$-ideals of A. If $J(A) = \{0\}$, A is J-semisimple. THEOREM 1: J(A) is an $\mathscr{l}$-QR $\mathscr{l}$-ideal which contains every $\mathscr{l}$-QR one-sided $\mathscr{l}$-ideal of A. THEOREM 2: J(A) is the intersection of all $\mathscr{l}$-primitive $\mathscr{l}$-ideals of A. (Received December 1, 1958.)

553-65. M. S. Klamkin: One-dimensional heat flow with space-varying thermal properties.

The one-dimensional heat flow equation $c\rho\, \partial T/\partial t = (\partial/\partial x)\{k\, \partial T/\partial x\}$ is solved analytically for thermal properties given by $\alpha(x) = m^{-2}F'(x)^{-2}$ and $k(x) = aF'(x)^{-1}$ (a and m are constants and F(x) is arbitrary) and initial and boundary conditions by $T(x,0) = 0$, $\partial T/\partial x]_{x=0} = 0$, $k\,\partial T/\partial x]_{x=L} = Q$. This solution is related to the corresponding one with constant thermal properties. In addition, two other analytic solutions are derived but in these cases for a semi-infinite bar. Here, $\alpha =$ constant, $k = k_0 e^{-2bx}$; $\alpha = (a + bx)^2$, $k =$ constant. These latter two solutions are interrelated. (Received December 1, 1958.)

553-66. J. W. Neuberger: On the existence of certain Green's functions.

Suppose that (1) S is a normed, abelian and complete space and B the associated space of all bounded linear transformations from S to S, (2) [a,b] a number interval and each of F and H a function from [a,b] to B of bounded variation on [a,b] with F continuous, and (3) M the function from $[a,b] \times [a,b]$ to B such that $M(x,t) = I + \int_t^x dF \cdot M(j,t)$ for each of x and t in [a,b] with I the identity in B. If for some t in [a,b], $\int_a^b dH \cdot M(j,t)$ has a bounded inverse which is from S onto S, there is a continuous function R from [a,b] to B and a function K from $[a,b] \times [a,b]$ to B such that if g is a continuous function from [a,b] to S and C is in S, the only continuous function X from [a,b] to S such that $X(t) = X(u) + g(t) - g(u) + \int_u^t dF \cdot X$ and $\int_a^b dH \cdot X = C$ for each of t and u in [a,b] is given by $X(t) = R(t)C + \int_a^b K(t,j)dg$ for each t in [a,b]. Moreover, such a pair of

functions R and K is given by $R(t) = [\int_a^b dH \cdot M(j,t)]^{-1}$ and $K(t,u) = -[\int_a^b dH \cdot M(j,t)]^{-1} \int_u^b dH \cdot M(j,u) + M(t,u)$ if $a \leqq u \leqq t$ and $= -[\int_a^b dH \cdot M(j,t)]^{-1} \int_u^b dH \cdot M(j,u)$ if $t < u \leqq b$. (Received December 1, 1958.)

553-67. E. T. Parker: Construction of some sets of pairwise orthogonal latin squares. Preliminary report.

For each integer $n > 1$ there exists a set of t pairwise orthogonal latin squares (p.o.l.s.) of order n, where t is one less than the smallest factor of the prime-power decomposition of n (Mann, Analysis and design of experiments, Dover, 1949, p. 105). It seemed plausible that t is the maximum; this would have implied that all finite projective planes are of prime-power orders, and Euler's conjecture that no pair of orthogonal latin squares exists for $n \equiv 2 \pmod 4$. This note establishes for some orders (perhaps infinitely many) the existence of larger sets of p.o.l.s. Theorem: If there exists a balanced incomplete block design with $\lambda = 1$ and k a prime-power, then there exists a set of $k - 2$ p.o.l.s. of order v. All blocks permuted by a doubly transitive group of order $k(k - 1)$, together with k-tuples of a repeated symbol, coordinatize $k - 2$ p.o.l.s. The author has obtained more than t p.o.l.s. as given by the Corollary: If m is a Mersenne prime > 3 or $m + 1$ is a Fermat prime > 3, then there exists a set of $m - 1$ p.o.l.s. of order $m^2 + m + 1$. In these cases $t = 2$. The block design applied is the projective plane of order m. (Received December 1, 1958.)

553-68. Pasquale Porcelli: Weak convergence and weak compactness in the space of functions of bounded variation.

A necessary and sufficient condition is derived in order that a sequence $\{f_n\}_{n=1}^{\infty}$ in B[a,b] be weakly convergent. This condition is used to obtain a necessary and sufficient condition in order that a subset of B[a,b] be sequentially weakly compact. These conditions are applied to the space R (which consists of functions f(x), analytic for $|x| < 1$, and such that $f(x) = \sum_{i=1}^{n} a_i f_i(x)$, where a_i is real and $f_i(x)$ is analytic and has positive real part for $|x| < 1$ and obtain similar results. An example is given of a weakly convergent sequence $\{f_n\}_{n=1}^{\infty}$ of singular continuous function in B[a,b] which is not norm convergent. This example contradicts a previously announced result by the author. (Received December 1, 1958.)

553-69. C. H. Wilcox: Spherical means and radiation conditions.

Let $u(p)$ solve $\Delta u + k^2 u = 0$ in an exterior domain V. Then the spherical mean $\tilde{u}(p,r)$ of $u(p)$ for center p, radius r is defined for all p and $r > r_0(p)$ and has the form $\tilde{u}(p,r) = ie^{-ikr}u_1(p)/2kr - ie^{ikr}u_2(p)/2kr$ where $u_1(p)$ and $u_2(p)$ are entire solutions of $\Delta u + k^2 u = 0$. If $u(p)$ is called a radiation function when $u_1(p) = 0$, then $u(p) = u_1(p) + v_1(p)$ where $u_1(p)$ is entire, $v_1(p)$ is a radiation function and the decomposition is unique. The condition $u_1(p) \equiv 0$ is shown to be equivalent to Sommerfeld's radiation condition $\lim_{r\to\infty} \int_{S(0,r)} |\partial u/\partial r - iku|^2 dS = 0$. If $\phi(p,t)$ solves the initial-boundary value problem (1) $\Delta\phi - \partial^2\phi/\partial t^2 = f(p,k)e^{-ikt}$ in V, $t > 0$, where f vanishes outside of a sphere, (2) $\phi(p,0+) = 0$, $\partial\phi(p,0+)/\partial t = 0$ in V (3) $\phi(p,t) = 0$ or $\partial\phi(p,t)/\partial n = 0$ on ∂V, $t \geq 0$, it is shown that $\phi(p,t) = u(p,t) + e^{-ikt}u(p)$ where $u(p,t)$ is a transient; i.e., $\lim_{t\to\infty} u(p,t) = 0$, and $u(p)$ is independent of t and satisfies (a) $\Delta u + k^2 u = f$(b) $u(p) = 0$ or $\partial u(p)/\partial n = 0$ on ∂V. Moreover, it is shown that $u(p)$ is necessarily a radiation function. Thus, by Rellich's uniqueness theorem (Jber. Deutschen Math. Verein. vol. 53 (1943) pp. 57-64) the steady-state part $u(p)$ of $\phi(p,t)$ is characterized by (a) and (b) and Sommerfeld's radiation condition. Analogous results are derived for solutions of Maxwell's equations. (Received December 1, 1958.)

553-70. R. H. Bruck: Some theorems on Moufang loops.

The paper investigates several loosely allied topics on loop theory with special reference to Moufang loops. Some of the end results are as follows, where G is a Moufang loop, N is the nucleus of G, and G^3 is the subloop generated by all cubes of elements of G: (1) If $G = NG^3$, every loop isotopic to G is isomorphic to G. (2) If $G \neq 1$ is simple and if $G^3 = 1$, G is the group of three elements. (3) If G is simple, $G = NG^3$. Here (1) is a corollary of: (4) NG^3 is a characteristic normal subloop of G, and every element of NG^3 is a companion of at least one pseudo-automorphism of G. Moreover, (3) is a consequence of (4) and (2). If Z and G' are the centre and commutator-associator subloop of G, (2) is a corollary of the following "Burnside" theorem: (5) If G satisfies the identity $(xy)^3 = x^3y^3$, then (a) every composition chief system of G is a central system of G; (b) G' and G/Z have exponent 3 and are locally finite, locally centrally nilpotent; (c) G is locally centrally nilpotent and, if periodic, is locally finite. (Received December 2, 1958.)

553-71. Lee Lorch: The Lebesgue constants for Jacobi series.

For $\alpha > -1$, $\beta > -1$, let $L_n(\alpha,\beta)$ denote the n-th Lebesgue constant (norm) associated with developments in terms of Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ at the end-point $x = 1$. [At the other end-point, $x = -1$, the corresponding constant reduces to $L_n(\beta,\alpha)$.] For $\alpha > -1/2$, H. Rau proved (Crelle's Journal, vol. 161 (1929) pp. 237-254) that $L_n(\alpha,\beta) = A(\alpha,\beta)n^{\alpha+1/2} + O(n^{\alpha+1/2})$, and determined $A(\alpha,\beta)$ explicitly in terms of α and β. Here (1) Rau's error term is improved and the following results established: (2) $L_n(-1/2,\beta) = (4/\pi^2)\log n + C(\beta) + O(n^{-1}\log n) + O(n^{-\beta-3/2})$; (3) If $-1/2 < \alpha < 1/2$ and $\alpha - \beta < 1$, then $L_n(\alpha,\beta) = A(\alpha,\beta)n^{\alpha+1/2} + B(\alpha) + O(n^{\alpha-1/2}) + O(n^{\alpha-\beta-1})$, where $C(\beta)$ and $B(\alpha)$ are explicitly determined constants depending only on the parameters indicated. The proof of (1) uses Rau's representation of $L_n(\alpha,\beta)$. The proofs of (2) and (3) use a different representation, namely, $L_n(\alpha,\beta) = [N^{\alpha+1}/\Gamma(\alpha+1)] \cdot \int_0^{\pi} [\sin(\theta/2)]^{\alpha-1/2}[\cos(\theta/2)]^{\beta+1/2}(\theta/2)^{1/2}|J_{\alpha+1}(N\theta)|d\theta + O(n^{\alpha-1/2}) + O(n^{\alpha-\beta-1})$, where $J_{\alpha+1}$ is the Bessel function of the first kind and $N = n + (\alpha+\beta+2)/2$. The case $\alpha = \beta = -1/2$ is equivalent to ordinary Fourier series, and $\alpha = \beta = 0$ to Laplace series. (Received December 2, 1958.)

553-72. C. H. Cunkle and W. R. Utz: Equicontinuous and related flows.

A flow is a topological semigroup S of continuous transformations of a metric space X into itself. In the presence of the equicontinuity of S, certain recurrence properties follow from weaker (nonwandering) properties, and several recurrence properties on X follow from the corresponding properties on a dense subset K of X. In case S is isomorphic to the additive group of reals, the hypothesis of equicontinuity may be weakened. When S is a group, S is shown to be weakly transitive on an orbit closure if it is equicontinuous at a point of the orbit closure and strongly transitive if it is equicontinuous at every point of the orbit closure. From this follows the theorem that under an equicontinuous flow minimal sets are the same as orbit closures. In the case of the additive group of reals equiuniform continuity is characterized by a weaker hypothesis. (Received December 2, 1958.)

553-73. B. Grunbaum: Intersections of cells in normed spaces.

For a real normed space X with unit cell S let E(X) denote the g.l.b. of reals c having the property: Given any family of mutually intersecting cells $\{x_i + \alpha_i S;\ i \in I\}$, the family $\{x_i + c\alpha_i S;\ i \in I\}$ has a nonvoid intersection. E(X)

is called exact if $\bigcap_{(i\in I)} x_i + E(X)\alpha_i S \neq \emptyset$. Then: (i) For uncomplete spaces $E(X) = \infty$; for Banach spaces $E(X) \leqq 2$. (ii) $E(X)$ is exact for adjoint Banach spaces, but also for some nonadjoint spaces such as (c_0). (iii) For n-dimensional spaces $E(X) \leqq 2n/(n+1)$; those with $E(X) = 2n/(n+1)$ are characterized. (iv) For Euclidean spaces $E(R^n) = [2n/(n+1)]^{1/2}$. (v) There exist spaces, even two-dimensional ones, for which $E(X)$ is greater than the diameter of the minimal cell which may cover any set of diameter 1. The results generalize those of Bohnenblust [Ann. of Math. vol. 39 (1938) pp. 301-308], Leichtweiss [Math. Z. vol. 62 (1955) pp. 37-49] and Jung [J. Reine Angew. Math. vol. 123 (1901) pp. 241-257]. Applications of $E(X)$ to problems of projections and extensions of transformations in Banach spaces, as well as generalizations to metric spaces, will be considered in another communication. (Received December 2, 1958.)

553-74. S. R. Mandan and Seymour Schuster: Some properties of certain pencils of conics.

The relation of the common points of triangular and pentagonal pencils of conics (see Schuster, Pencils of polarities in projective space, Canad. J. Math. vol. 8 (1956) pp. 119-144) to their defining configurations are determined. In addition, affine and euclidean interpretations are made of the two pentagonal pencils, so that the nature of the conics is determined for all variations in the self-polar pentagon and its related points. (Received December 2, 1958.)

553-75. Paromita Chowla: Diophantine equations in quadratic number-fields.

Let $a^2 - 4b$ be square-free and less than -3, and let (*) $s_n + as_{n-1} + bs_{n-2} = 0$ $(n \geqq 3)$ (where a,b like s_1, s_2 are fixed but arbitrary rational integers) determine s_n for all natural numbers n. It is an unsolved problem to find an explicit function $f = f(a,b,c,s_1,s_2)$ such that the equation $s_n = c$ has no solutions for $n > f$ [naturally one can extend our problem to the case when the 3-term relation in (*) is replaced by one with h terms $(h > 3)$]. For example, one can ask whether (provided $a^2 - 4b$ is square-free and less than -3) $f = 10^{10w}$, $w = 6 + |a| + |b| + |c| + |s_1| + |s_2|$ is an admissible value of f. Following S. Chowla, D. J. Lewis, and Th. Skolem, who have solved special cases, the author finds f when $s_1 = 1$, $s_2 = -1$, $a = -2$, $b = 3$, $c = 1$. Result: $f = 5$. (Received November 24, 1958.)

553-76. W. R. Baum: Integral geometric methods in information theory. I. Invariant measures.

As a first step toward a multidimensional generalization of information theory motion invariant measures of certain sets of geometric elements in Euclidean n-space E_n are introduced. In particular, if the geometric elements are linear subspaces E_i, ($i = 0,1,\ldots,n$), of E_n and if C_α, ($\alpha \in J$; J index set), are bounded closed convex point sets in E_n, the E_i-measures of C_α lead to the consideration of Minkowski's cross measure integrals of C_α and their rôle in the probability space representing the "source". Relations to Hadwiger's theorem (H. Hadwiger, Abh. Math. Sem. Univ. Hamburg vol. 17 (1951) pp. 69-76) about representability of a motion invariant, additive and monotone functional on the C_α's in terms of the i-dimensional contents of the C_α's are studied. Generalizing, one considers the geometric elements as elements of a homogeneous space of a (simply) transitive Lie group and obtains invariant measures expressible, as shown by S. S. Chern (Ann. of Math. vol. 43 (1942) pp. 178-189), by integrals of exterior products which are "densities" of geometric elements. [Sponsored by the Information Systems Branch of ONR (NONR-669 (10)).] (Received December 3, 1958.)

553-77. V. E. Beneš: General stochastic processes in models of telephone traffic and Type I counters.

The condition of N telephone trunks used in a fixed order of preference, or of N Type I counters arranged in cascade, can be described by a set of coupled nonlinear stochastic integral equations. The preceding statement holds for any physically reasonable choice of stochastic process to describe the arrivals and holding-times (or dead times). If holding-times form a renewal process, and arrivals are in an (independent) general process $\Lambda(t)$ = number of arrivals in $(0,t)$, the condition of the N trunks, relative to knowledge of $\Lambda(\cdot)$, can be described by stochastic integral equations of Kolmogorov type. Under a weak stationarity assumption these equations can be attacked by methods of renewal theory. (Received December 3, 1958.)

553-78. J. W. Brace: Almost uniform convergence versus pointwise convergence.

This paper determines that, under reasonable conditions, almost uniform convergence of a net of functions (see author, Portugaliae Math. vol. 14 (1955)

pp. 99-104) is equivalent to pointwise convergence of a net of extended functions over an enlargement of the domain. It is shown that there is a "best" enlargement of the domain. If the original domain of the functions is given the coarsest topology to make all the functions under consideration continuous, then the enlarging process presented in the paper can be viewed as a compactification of the domain and is the smallest compactification to which the functions can be continuously extended. (Received December 3, 1958.)

553-79. William Browder: Suspensions in loop spaces.

Let G be the space of loops of a space, and suppose G is connected and simply connected, (for instance 1-connected topological groups, up to homotopy type). Let p be an odd prime. Suppose $x \in H_{2q}(G,Z_p)$ and $x = \sigma y$, where σ is the homology suspension σ: $H_{2q-1}(\Omega G;Z_p) \to H_{2q}(G;Z_p)$ and $y \in H_{2q-1}(\Omega G;Z_p)$. Let $\bar{x} \in H^{2q}(G;Z_p) = \mathrm{Hom}(H_{2q}(G;Z_p), Z_p)$ such that $\bar{x}(x) \neq 0$. Theorem 1. The elements $x^p \in H_{2qp}(G;Z_p)$ and $\bar{x}^p \in H^{2qp}(G;Z_p)$ cannot both be zero. The proof involves homology operations defined by the author (see Abstract 547-35 Notices Amer. Math. Soc. vol. 5 (1958) p. 461). Some simple consequences of this theorem are: Theorem 2. The tensor product of an exterior algebra and a truncated polynomial ring is not realizable as the homology ring with coefficients Z_p of a 1-connected loop space. Theorem 3. If G is (2n-1)-connected then $\pi_{2n}(G)$ has no p-torsion if $H_{2np^2(p-1)}(G;Z_p) = 0$. Corollary. If G is (2n - 1)-connected then $\pi_{2n}(G)$ has order a power of two if $H_m(G) = 0$ for $m \geq 36n - 1$. (Received December 3, 1958.)

553-80. Leon Brown, A. L. Shields and Karl Zeller: On absolutely convergent exponential sums.

J. Wolff [C. R. Acad. Sci. Paris vol. 173 (1921) p. 1327] gave a representation of zero by an absolutely convergent series of exponentials with bounded exponents: $\sum_1^\infty a_n e^{\alpha_n z} \equiv 0$, $0 < \sum_1^\infty |a_n| < \infty$, $|\alpha_n| \leq M$, $n = 1,2,\ldots$. In this paper, some problems related to Wolff's results are considered. Under certain sufficient conditions (for example, the exponents are all real) this representation does not exist. The case where the exponents $\{\alpha_n\}$ lie inside the unit circle, but have no interior limit point, is considered. Under this condition, the following are equivalent: (1) zero may be represented, (2) every entire function of exponential type less than one may be represented (3) $\sup_n |f(\alpha_n)| = \sup_{|z|<1} |f(z)|$ for all bounded analytic functions f (denote this space of functions by H_∞),

(4) there exists an interpolation formula, i.e., there exists (a_n), $\sum_1^\infty |a_n| < +\infty$ such that $f \in H_\infty$, then $f(0) = \sum a_n f(\alpha_n)$. A geometric characterization of the sets $\{\alpha_n\}$ for which the above is possible is given. With a suitable norm the set of functions represented by $\sum_1^\infty a_n e^{\alpha_n z}$ forms a Banach space E. The relationship between E and other spaces of analytic functions is discussed. (Received December 3, 1958.)

553-81. Y. W. Chen: Hoelder continuity and initial value problems of mixed type differential equations.

Consider the equation $x^m u_{yy} + u_{xx} = 0$ with $m > -1$ in a domain D in the half-plane $x > 0$ with a boundary segment on $x = 0$. Let the initial data be $u(0,y) = u_0(y)$, $u_x(0,y) = u_1(y)$. The purpose of this paper is (1) to give a characterization of the initial data in terms of initial values of analytic functions of a complex variable W in D, and (2) to prove Hoelder continuity properties of u_0 and u_1 in a theorem which is analogous to Provaloff's theorem on conjugate harmonic functions. For $m = p > 0$, let $\alpha = p(2p + 4)^{-1}$ and $1 - \alpha < \mu < 1$. (1). To each solution u of class C_2 in D with $u_0 \in H_{\mu-\alpha}$ (Hoelder exponent $\mu - \alpha$) and $u_1 \in H_{\mu+\alpha-1}$ there is an analytic function g(W) in D such that $|y|^\alpha g(y) \in H_\mu$, and conversely. Let $I^\alpha(f)$ and $I^{-\alpha}(f)$ denote the integration and differentiation of fractional order α of a function f, and introduce $I_\beta^\alpha = s^\beta I^\alpha(s^{-\beta} f(s))$, $I_\beta^{-\alpha} = s^\beta I^{-\alpha}(s^{-\beta} f(s))$. Then (u_0, u_1) are related to (g_0, g_1) where $g(y) = g_0 + i g_1(y)$, by the following formula: $s^\alpha[g_0(\pm s) + \tan \alpha\pi g_1(\pm s)] = A_0 I_{1-\alpha}^\alpha u_0(\pm s)$, $s^\alpha g_1(\pm s) = \mp A_1 I_\alpha^{1-\alpha} u_1(\pm s)$ where $s = |y|$ and A_0, A_1 are constants. (2). $u_0 \in H_{\mu-\alpha}$ implies the existence of u_1 and $u_1 \in H_{\mu+\alpha-1}$, and conversely. Similar results are obtained for other values of μ and for equations with $m = q$, $-1 > q > 0$. (Received December 3, 1958.)

553-82. W. V. Caldwell: Maximal vector spaces of light, interior, orientation-preserving C' functions.

Let $\mathcal{D}$ be a domain in E^2 and let $C \subset \mathcal{D}$ be a compact, zero-dimensional set. Let a(x,y), b(x,y), c(x,y), and d(x,y) be Hölder-continuous, uniformly bounded functions in $\mathcal{D}$ such that $4bc \geqq (a + d)^2$, $b > 0$, where equality holds at most on C. A function $f(x,y) = u(x,y) + iv(x,y)$ will be said to be a solution of the system of partial differential equations (1) $U_x = aV_x + bV_y$, $-U_y = cV_x + dV_y$ if the pair u,v satisfy (1). Now let A be the set of functions mapping $\mathcal{D}$ into E^2 which are light, interior, C', and orientation-preserving and

let $\mathscr{V}$ be a real linear vector space of elements of A. It is proved that if two linearly independent elements of $\mathscr{V}$ are solutions of (1), then every other element of $\mathscr{V}$ is also a solution of (1). This theorem is a generalization of a theorem due to C. J. Titus and J. E. McLaughlin in their paper A characterization of analytic functions, Proc. Amer. Math. Soc. vol. 5 (1954) pp. 348-351. (Received December 3, 1958.)

553-83. H. S. M. Coxeter: Close-packing and froth.

It was observed by E. B. Matzke [Bull. Torrey Botanical Club, vol. 77 (1950) pp. 223-225] that if lead shot are poured into a cylinder and compressed with a plunger at a pressure of about 40,000 pounds, they become irregular polyhedra most of which have 14 faces. Also, in a froth of approximately equal bubbles, the average number of contacts of a bubble is about 13.7. The two-dimensional analogue of either experiment obviously yields the exact number 6: the plane is filled with a regular tessellation $\{6,3\}$ consisting of hexagons, three at each vertex. The reason for the irregularity in three dimensions is that the theoretical regular honeycomb $\{p,3,3\}$ (consisting of polyhedra $\{p,3\}$, three at each edge) has a value of p between 5 and 6, and so exists only in a statistical sense. A formula due to Steinberg [Canad. J. Math. vol. 10 (1958) p. 220] shows that a regular honeycomb $\{p,q,r\}$ satisfies the equation $p - (4/p) + 2q + r - (4/r) = 12$. When $q = r = 3$, this yields $p = (13 + 313^{1/2})/6 = 5.115\ldots$. It follows that the number of faces of the cell $\{p,3\}$ is $12/(6 - p) = (23 + 313^{1/2})/3 = 13.56\ldots$. (Received December 3, 1958.)

553-84. R. J. Dickson: A model of turbulence displaying autonomous oscillations. Preliminary report.

The system A: $u_t = \nu u_{xx} + (1 + i)u - 2\bar{u}\bar{u}_x$, $\nu = \text{const.} > 0$, $u(x,t) = v(x,t) + iw(x,t)$, $0 \leq x \leq 1$, $t \geq 0$, $u(0,t) = u(1,t) = 0$, is one model of turbulence proposed by J. M. Burgers (Kon. Nederl. Akad. Wetensch.Verslagen, Afd. Natuurk vol. 17 (1939) pp. 1-53), who conjectured the existence of oscillatory solutions in the "turbulent regime," $\nu < \pi^{-2}$, for which the "laminar" solution $u = 0$ is unstable. A key feature of this model is the relation B: $\mathrm{Im}\int_0^1 \bar{u}u_t dx = \int_0^1 |u|^2 dx$, which precludes nonvanishing time-independent solutions. As simpler analogous models this paper proposes the systems of o. d. e.'s obtained by discretizing the x-dependence of A as follows: $u_k = \nu n^2(u_{k+1} - 2u_k + u_{k-1}) + (1 + i)u_k - (n/3)(\bar{u}_{k-1} + \bar{u}_k + \bar{u}_{k+1})(\bar{u}_{k+1} - \bar{u}_{k-1})$, $u_k = u_k(t) = v_k(t) + iw_k(t)$, k

$= 1,2,\ldots,n-1$, $u_0 = u_n = 0$, yielding $\mathrm{Im}\sum \bar{u}_k \dot{u}_k = \sum |u_k|^2$ as an analog of B. For $\nu > \nu_0 = (2n \sin(\pi/2n))^{-2}$ every solution approaches the trivial (laminar) solution $u_k = 0$. Main Result: In the case $n = 3$ (fourth order real autonomous system) there is a turbulent regime, $\nu_1 < \nu < \nu_0$, in which all nontrivial solutions tend exponentially to a unique periodic solution. (Received December 3, 1958.)

553-85. W. B. Fulks and J. S. Maybee: On a nonlinear parabolic equation.

Let Δ be the Laplace operator in E^m and let $Hu = u_t - \Delta u$ be the heat operator on u. Let R be a fixed bounded domain in E^m regular for Laplace's equation and let $F(x,t,u)$ be a function such that (i) F is defined and non-negative for $x \in R$, $t \geq 0$ and $u > 0$, (ii) $F(x,t,u)$ is locally Hölder continuous in all arguments and bounded if $u \geq c > 0$, (iii) $F(x,t,u)$ is nonincreasing in u with bounded difference quotient if $u,v \geqq c > 0$. $F(x,t,u)$ could be, for example, of the form u^{-k} as a function of u. The equation $Hu = F(x,t,u)$ is studied. R is called regular for this equation if the Dirichlet problem for the space-time cylinder with R as base has a unique non-negative solution for each given non-negative continuous function on the boundary. Regularity is similarly defined for the corresponding elliptic equation $\Delta u + F_\infty(x,u) = 0$ where $F_\infty(x,u)$ has properties analogous to those of $F(x,t,u)$. It is shown that if R is a regular region for Laplace's equation it is also for $Hu = F(x,t,u)$. It is also shown, by considering the steady state solutions, that R is regular for the corresponding elliptic equation. This is done under the assumption that $F(x,t,u)$ tends uniformly for $x \in R$ to $F_\infty(x,\bar{u})$ as t tends to infinity and u to $\bar{u}$. (Received December 3, 1958.)

553-86. Melvin Henriksen: Multiplicative summability methods and the Stone-Čech compactification.

A linear functional ϕ defined on a subspace $\mathscr{D}^*(\phi)$ of the space $C^*(N)$ of bounded sequences of real numbers, such that $\phi(f) = a$ whenever f converges to a, is called a regular summability method. If, in addition, $\phi(fg) = \phi(f)\,\phi(g)$ for all f,g in $\mathscr{D}^*(\phi)$, we call ϕ an M-method. Let βN denote the Stone-Čech compactification of the space N of positive integers, and let $\bar{f}$ denote the continuous extension over βN of $f \in C^*(N)$. THEOREM 1. For every M-method ϕ there is a (unique) closed subset $F(\phi)$ of $\beta N - N$ such that $\phi(f) = a$ iff $\bar{f}[F(\phi)] = a$. Conversely, if $F(\phi)$ is a closed subset of $\beta N - N$, and we let $\phi(f) = a$ whenever $\bar{f}[F(\phi)] = a$, then ϕ is an M-method. Corollary 1. If $\mathscr{D}^*(\phi) \neq C^*(N)$, then $\mathscr{D}^*(\phi)$

determines ϕ. Corollary 2. Every M-method ϕ is determined by the sequences of 0's and 1's in $\mathscr{S}^*(\phi)$. THEOREM 2. If $F(\phi)$ is the closure of a countable union of open and closed subsets of $\beta N - N$, then the M-method ϕ is equivalent (for bounded sequences) to a regular matrix summability method. Hence not every matrix M-method is equivalent to a submethod of the identity. This answers a question posed in Amer. Math. Monthly vol. 63 (1956) pp. 323-326. (Received December 3, 1958.)

553-87. Edwin Hewitt and H. S. Zuckerman: Some theorems on lacunary Fourier series, with extensions to compact groups.

Let N be a set of integers. The following properties of N are equivalent. (i) There is a positive number A such that for every finite sequence $\{a_1,\dots,a_s\}$ of complex numbers and every finite subset $\{n_1,\dots,n_s\}$ of N, the inequality $\sum_{k=1}^{s}|a_k| \le A\cdot\max_{0\le t<2\pi}\left|\sum_{k=1}^{s}a_k e^{in_k t}\right|$ obtains. (ii) If $x \in \mathfrak{L}_\infty(0,2\pi)$ and the Fourier coefficients of f vanish off of N, then $\sum_{n\in N}|x(n)| < \infty$. (iii) For every function ϕ on N that is arbitrarily small in absolute value outside of finite sets, there is a function $x \in \mathfrak{L}_1(0,2\pi)$ such that $x(-n) = \phi(n)$ for $n \in N$. (iv) For every bounded function on N, there is a Radon measure μ on $[0,2\pi]$ such that $\mu(-n) = f(n)$ for all $n \in N$. An analogous theorem giving conditions under which continuous periodic functions map onto $\mathfrak{l}_2(N)$ by the Fourier transform is also proved. Both theorems have extensions to arbitrary compact groups. Some sets are found that satisfy the conditions of the first theorem and are not the union of any finite family of sets lacunary in the sense of Sidon and Banach. The set of all numbers $3^{2^{l+2}} + 3^k$, $k = 2^l, 2^l+1, 2^l+2,\dots,2^{l+1}-1$, and $l = 0,1,2,\dots$, is such a set. (Received December 3, 1958.)

553-88. H. B. Jenkins: Dual variational problems.

Let $\phi(x,y)$ be a solution of the quasi-linear elliptic equation $F_{uu}\phi_{xx} + 2F_{uv}\phi_{xy} + F_{vv}\phi_{yy} = 0$; arising from the variational problem $\delta\int F(u,v)dxdy = 0$, where $u = \phi_x$, $v = \phi_y$, and $F_{uu}F_{vv} - F_{uv}^2 > 0$. Then $d\xi = F - vF_v dx + vF_u dy$, $d\eta = uF_v dx + F - uF_u dy$ are exact differentials in the domain of ϕ. If the surface $w = F(u,v)$ is tangent to a cone at infinity, the transformations $\xi(x,y)$, $\eta(x,y)$; $\sigma = \phi_\xi = u/F$, $\tau = \phi_\eta = v/F$ carry ϕ by invariance to a solution $\phi(\xi,\eta)$ of a "dual" variational problem $\delta\int G(\sigma,\tau)d\xi d\eta = 0$ where $G(\sigma,\tau) = \{F(u(\sigma,\tau), v(\sigma,\tau))\}^{-1}$. If in addition it is assumed that the original equation is of minimal surface type in the sense of Finn, then the dual equation is uni-

formly elliptic, and estimates of the second derivatives are obtained which are analogous to those obtained by E. Heinz for minimal surfaces. Bernstein's theorem on the nonexistence of nonlinear entire solutions is a corollary. Such equations also have the property that the gradient of a solution defined in a neighborhood of infinity has a finite limit at infinity independent of direction. If F is a function of $q = (u^2 + v^2)^{1/2}$ only, Bernstein's theorem is a consequence of the condition of tangency to a cone, and a monoticity condition on the second derivative of F. (Received December 3, 1958.)

553-89. J. E. Keisler: On generalized Vietoris complexes and nerve complexes. Preliminary report.

Let S be a finite collection of nonempty sub-sets of X; N(S), the nerve of S; and V(S), the generalized Vietoris complex of X. That is, (n + 1) points of X are an n-simplex of V(S) if the (n + 1) points are contained in some element of S. Only finite chains on V(S) are considered. In a manner analogous to the definition of barycentric subdivision maps, chain maps ϕ^S and θ^S are defined from the chain group C(N(S),G) to C(V(S),G) and from C(V(S),G) to C(N(S),G), respectively. By constructing a chain homotopy operator between ϕ^S and θ^S, it is shown that ϕ^S_* and θ^S_* are inverse isomorphisms between the homology groups H(N(S),G) and H(V(S),G). Let W be a finite collection of nonempty sub-sets of X which refines S; π, a projection chain map of N(W) into N(S); and i, the mapping of V(W) into V(S) defined by $i(x_0 \ldots x_n) = (x_0 \ldots x_n)$. Then $\theta^S i \sim \pi \theta^W$ and $\phi^S \pi \sim i \phi^W$. Thus, these results yield another proof that the Čech and the generalized Vietoris homologies of a compact space agree, show that Lefschetz' quasi-complexes may be defined in terms of generalized Vietoris complexes, etc. (Received December 3, 1958.)

553-90. Paolo Lanzano: Projective-geodesic mapping of surfaces. Preliminary report.

In the theory of surfaces imbedded in a three-dimensional projective space Fubini's projective element of arc is the ratio F_3/F_2 of a cubic and quadratic differential form where $F_3(u,v; du,dv) = 0$, $F_2(u,v; du,dv) = 0$ represent the differential equations of the Darboux and asymptotic curves respectively. A projective-geodesic mapping of a surface onto another is a mapping preserving the extremals (pangeodesics) of Fubini's form. The following results were obtained: if there exists a projective-geodesic mapping between two surfaces only

three cases can occur: (1) the projective elements are proportional by a constant factor, (2) the surfaces are ruled and their generators are corresponding lines, (3) the equations of their pangeodesics are divisible by one or more factors of F_3. In the last two cases appropriate forms for the projective elements were obtained. (Received December 3, 1958.)

553-91. L. F. Markus: Structurally stable differential equations.

A differential system is structurally stable in case a small (in C^1-topology) perturbation of the coefficients leaves the qualitative form of the solutions unchanged. A structurally stable differential system has isolated elementary critical points and periodic solutions. A minimal compact invariant set, which is not a critical point, must be approached by a sequence of periodic solutions. (Received December 3, 1958.)

553-92. R. J. Mihalek: Modularity relations in lattices.

Let L be a lattice with operations $+, \cdot$, and ordering $\leqq$. For $R \subset T \subset L \times L$, R is a modularity relation under T if (i) $(b,c) \in R$, $b' \leqq b$, $c' \leqq c$, $bc \leqq b'$, c', $(b',c') \in T$ implies $(b',c') \in R$; (ii) $(c,d) \in R$, $(b,c+d) \in R$, $b(c+d) = cd$ implies $(b+c,d) \in R$, $(b+c)d = cd$; and R satisfies the intersection property if $(b+d)(c+d) = d$ is added to the conclusion of (ii). For such an R, an independence theory is obtained comparable to the theory for ordinary independence (L. R. Wilcox, Modularity in the theory of lattices, Ann. of Math., April, 1939). In the theory of quasi-dual-ideals, the concept of a quasi-modularity relation is defined in terms of the above. Instances of this abstract modularity and quasi-modularity are the relations considered in the preliminary report (Notices Amer. Math. Soc. vol. 5 (1958) p. 349). (Received December 3, 1958.)

553-93. J. A. Nohel: A nonlinear delay-differential equation.

Sufficient conditions for stability in the sense of Liapounov for systems of nonlinear differential equations were extended to particular difference-differential equations by several authors; e.g., R. Bellman and E. M. Wright. These results are generalized here to solutions of the functional equation (1) $u' = a\int_0^\theta (\theta - h)u(t-h)dh + \int_0^\theta (\theta - h)g[t-h, u(t-h)]dh$, $t > \theta$, $' = d/dt$, and $u(t) = q(t)$, $0 \leqq t \leqq \theta$. In (1) a, θ are real constants, $\theta > 0$, t, u, g, q, are real: q is a given continuous function and $g(t,u)$ is continuous for $t \geqq 0$, $|u|$ small, $g(t,0) = 0$. Notions of solutions and stability for (1) are defined. After discussing

questions of existence, uniqueness, and representation of solutions for associated linear inhomogeneous equations, stability is considered. One result is: If $a = -\omega^2/\theta$, $\omega \neq 2n\pi/\theta$, $n = \pm 1, \pm 2, \ldots$, and if $g(t,u) = o(|u|)$, $|u| \to 0$, uniformly in t, then the identically zero solution of (1) is asymptotically stable. The cases of $a < 0$, $\omega = 2n\pi/\theta$, and $a > 0$ are also considered. This generalizes earlier work of the author and W. K. Ergen (Stability of solutions of a nonlinear differential functional equation in reacta dynamics, unpublished Oak Ridge National Laboratory memorandum, 1955) where a special case (1) was studied. (Received December 3, 1958.)

553-94. F. P. Palermo: A universal coefficient system for cohomology rings.

The integral cohomology ring of a finitely generated cochain complex K is determined by the rational cohomology ring, the cohomology rings with integers modulo n as coefficients, for all torsion coefficients of K and the coefficient homomorphisms j and h_n induced by the inclusion of the integers into the rationals and the map which takes an integer into its residue class modulo n. The following lemma is used to prove the above result. Lemma: If $x \in H^p(K;Z)$ is such that $j(x) = 0$, $h_n(x) = 0$ for all torsion coefficients n, then $x = 0$. Another consequence of this lemma is the Theorem: If $F: K \to L$ is a cochain homomorphism, then the induced homomorphism $F_0: H^*(K;Z) \to H^*(L,Z)$ is determined by the maps $F_Q: H^*(K,Q) \to H^*(L,Q)$, $F_n: H^*(K,Z_n) \to H^*(L,Z_n)$ and the maps j and h_n for all torsion coefficients n of L. (Received December 3, 1958.)

553-95. G. M. Petersen: L-regular matrices.

A regular matrix $A = (a_{mn})$ is one which transforms convergent sequences into sequences convergent to the same sum. Almost convergent sequences have been defined by Lorentz to be sequences for which $\lim_{p \to \infty}(s_n + \ldots + s_{n+p})/p$ exists uniformly in n. In this note we investigate those matrices which are regular and which transform almost convergent sequences into sequences that are almost convergent to the same sum. The product of regular matrices is again a regular matrix, likewise the product of two L-regular matrices is again an L-regular matrix. Necessary and sufficient conditions for L-regularity are obtained. (Received December 3, 1958.)

553-96. B. E. Rhoades: Total comparison between two Cesàro matrices.

Let x denote a sequence, A an infinite matrix, and $A_n(x) = \sum_k a_{nk}x_k$. Let A and B be two infinite matrices. Then A is totally stronger than B (A t.s. B) if, for every sequence x with $B_n(x) \to \ell$, $A_n(x) \to \ell$, $|\ell| \leq \infty$. (See Basu, Proc. London Math. Soc. vol. 50 (1948-1949) pp. 447-462). For A and B Hausdorff matrices this reduces to showing that AB^{-1} is regular and is generated by a totally monotone sequence. Let C_α denote the Cesàro Hausdorff method of order α. THEOREM. Let $0 < \alpha < 1$. Then C_α t.s. $C_{-\alpha}$. The theorem is proved using the following two lemmas. LEMMA 1. Let μ be a non-negative sequence. If $\Delta\mu_k = \mu_k\alpha_k$, $k = 0,1,2,\ldots$, where α is a totally monotone sequence, then μ is totally monotone. LEMMA 2. Let H_μ, H_α be Hausdorff matrices with finite norm, H_α pure (Hausdorff, Math. Z. vol. 9 (1921) p. 75), and $\Delta\mu_k = c\alpha_k\mu_k$, c any constant for which $c\alpha_0 \neq 1$. Then H_μ is pure. The theorem cannot be extended for $\alpha \geq 1$. (Received December 3, 1958.)

553-97. R. D. Schafer: Nodal noncommutative Jordan algebras and simple Lie algebras of characteristic p.

L. A. Kokoris has recently determined all simple nodal noncommutative Jordan algebras A of characteristic $p \neq 2$ (Proc. Amer. Math. Soc. vol. 9 (1958) pp. 652-654; these Notices, vol. 5 (1958) p. 90). We establish relationships between these algebras A and certain recently discovered simple Lie algebras of characteristic p. The derivations of any A which is "defined by a skew-symmetric bilinear form ϕ" are determined, and an intrinsic characterization of the simple Lie algebras V_m of Albert and Frank (Rend. Sem. Mat. di Torino, vol. 14, pp. 117-139) is obtained: a Lie algebra L is a simple algebra V_m if and only if there is an A defined by a nondegenerate ϕ such that $L \cong D(A)'$ (the first derived algebra of the derivation algebra of A) in case $m > 1$, and $L \cong D(A)''$ in case $m = 1$. Each simple Lie algebra $L(G,\delta,f)$ of characteristic $\neq 2$ defined by Richard Block (Bull. Amer. Math. Soc. vol. 62 (1956) p. 349) can be obtained from a suitably chosen A: here the attached algebra A^- is a Lie algebra, and $L(G,\delta,f) \cong (\text{ad } A)'$, a simple ideal in D(A) (actually $L_0 \cong \text{ad } A$). (Received December 3, 1958.)

553-98. H. W. E. Schwerdtfeger: On groups where cosets are classes.

A group $\mathcal{G}$ with the elements a, b, ... is said to have the property T_1 if it contains a subgroup $\mathcal{H}$ such that for any pair of conjugate elements a and b

outside $\mathfrak{H}$ there is exactly one $h \in \mathfrak{H}$ which solves the equation $hah^{-1} = b$. If $\mathfrak{H}$ is a normal subgroup of $\mathfrak{G}$ and $\mathfrak{H}$ finite, then it is shown that the class of conjugate elements containing a coincides with the complete coset $\mathfrak{H}a$ and that $\mathfrak{H}$ is necessarily the commutator group of $\mathfrak{G}$; each of the elements of $\mathfrak{H}$ actually is a commutator and can be written in the form $hah^{-1}a^{-1}$ ($h \in \mathfrak{H}$). If $\mathfrak{H}$ is abelian then $\mathfrak{G}$ is metabelian. Given an additive group $\Gamma = (\alpha,\beta,\ldots)$ with a multiplicative abelian group $\mathfrak{A}$ of operators A, B,... acting in Γ, a T_1-group $\mathfrak{G}$ consists of all symbols (A, α), (B, β) with the multiplication rule $(A,\alpha)(B,\beta) = (AB, A\beta + \alpha)$. (Received December 3, 1958.)

553-99. H. S. Shapiro: On functions which satisfy rational recurrence formulae and related classes of real numbers. Preliminary report.

A k-tuple $\{\phi_1,\ldots,\phi_k\}$ of rational-valued functions defined on the positive integers is called R-recursive if there exist $R_1,\ldots R_k$, where each R_i denotes a quotient of two polynomials with integer coefficients, such that $\phi_i(n+1) = R_i[\phi_1(n),\ldots,\phi_k(n)]$, $i = 1,2,\ldots,k$. A rational-valued function is R-recursive if it is part of an R-recursive k-tuple, and the least possible k is called the order of $\phi(n)$. A real number α is R-recursive if there exists an R-recursive function $\phi(n)$ such that $\lim_{n\to\infty}\phi(n) = \alpha$, and the order of α is defined as the least order of all such ϕ. It is shown that the set E of R-recursive numbers is a field, and any real number algebraic over E is in E. R-recursive numbers of order one are the real algebraic numbers. Examples of R-recursive numbers are given, such as e^x, $e^{\pi x}$, when x is algebraic. Fragmentary results are obtained concerning a number of other apparently difficult questions, notably the nonemptiness of the classes of R-recursive numbers and functions of given order, the relation between R-recursiveness and computability, and a generalization of the concept of conjugates from algebraic to R-recursive numbers. (Received December 3, 1958.)

553-100. J. M. Shapiro: Some attraction properties of the Poisson distribution.

For $n = 1,2,\ldots$ let $S_n = x_{n1} + \ldots + x_{nk_n} - A_n$ be the sum of independent infinitesimal random variables minus suitably chosen constants A_n and let $F_n(x)$ be the distribution function of S_n. For $r > 0$ let $S_n^r = |x_{n1}|^r + \ldots + |x_{nk_n}|^r - B_n(r)$ where the $B_n(r)$ are suitably chosen constants and let $F_n^r(x)$ be the distribution function of S_n^r. In a previous paper by the author (Ann. of Math. Stat. vol.

29 (1958) pp. 515-522) the behavior of $F_n^r(x)$ was studied as $n \to \infty$ and $r \to \infty$. In this paper the case when $r \to 0^+$ is considered and in particular if $\lim_{n\to\infty} F_n(x) = F(x)$ then necessary and sufficient conditions for $\lim_{n\to\infty} F_n^r(x) = F^r(x)$ and $\lim_{r\to 0} F^r(x) = H(x)$, where $F(x)$, $F^r(x)$ and $H(x)$ are distribution functions, are obtained. It is shown that $H(x)$ is the sum of two independent random variables one Gaussian and the other Poisson including their degenerate cases, and in fact that the Poisson summand (nondegenerate) will be present in all but one special case, namely when $F(x)$ is Gaussian. (Received December 3, 1958.)

553-101. J. L. Snell and R. E. Williamson: Representation of martingales.

Under convenient restrictions on the density function $f(x)$ the authors represent the non-negative solutions $u(x,t)$, $-\infty < x < \infty$, $0 < t$, of the equation $u(x,t) = \int_{-\infty}^{\infty} u(x + y, t + 1)f(y)dy$. The representation kernel is expressed in terms of the bilateral Laplace transform of f. The result is the central one in a potential theory for the region $-\infty < x < \infty$, $t > 0$ and associated with the density f. An ideal boundary for the region, in the sense of R. S. Martin, is introduced, and the representation as well as a boundary limit theorem are interpreted in terms of this boundary. (Received December 3, 1958.)

553-102. D. E. Spencer: On the Lorenz potentials.

The statement is ordinarily made that the Lorenz potentials satisfy the scalar and vector wave equations and the Lorenz identity. Although this statement is valid for the majority of practical problems, the statement is not of general validity. Generalized scalar and vector wave equations are presented which are applicable to material media in arbitrary motion and are derivable from the Lorenz potentials. A fallacy in the customary derivation of the Lorenz identity is pointed out and its generalized form is given. (Received December 3, 1958.)

553-103. J. G. Stampfli: A spectral resolution for a bounded linear operator some power of which is normal.

Let T be a bounded linear operator on a Hilbert space H with T^N normal for some positive integer N. Then it is possible to reduce T to super diagonal form. More precisely, let $H' = \sum \oplus M_k$ where each $M_k = \int \oplus H_\lambda \, d\mu_k(\lambda)$. That

is, if $x \equiv \{x(\lambda)\} \in M_k$ then x is a function from the interval [a,b] into a Hilbert space of dim N. For $x, y \in M_k$ the inner product is defined as $(x,y) = \int(\{x(\lambda)\}, \{y(\lambda)\})d\mu_k(\lambda)$. (See Continuous direct sums of Hilbert spaces and some of their applications by M. A. Naimark and S. V. Fomin, Amer. Math. Soc. Translations, Ser. 2, vol. 5, 1957). Then H is isometric and isomorphic to H' and under this mapping T corresponds to T', with T' possessing the following properties: $(T'M_i, M_j) = 0$, for $i < j$ and for $T'|_{M_k}$ we have $T'x = T'\{x(\lambda)\} = \{t(\lambda)x(\lambda)\}$, where for each λ, $t(\lambda)$ is an N X N matrix in super diagonal form. If T is a bounded linear operator which commutes with a normal operator of finite spectral multiplicity, then we may obtain a similar resolution for T. (Received December 3, 1958.)

553-104. Diane Johnson and G. de B. Robinson: The reduction of an ordinary representation of S_n into its modular components for $p \leq n$.

The representations of S_p are p-integral as they stand and exhibit the reduction corresponding to Nakayama's decomposition matrices. This provides a start for the construction of a transforming matrix L, based on Young's raising operator in terms of which the decomposition matrices were determined by O. E. Taulbee (Thesis, Michigan State University) in the general case. Assuming the reduction for S_{n-1}, in which each modular component takes the same form, the reduction for S_n follows, thus proving the validity of Taulbee's construction. (Received December 3, 1958.)

553-105. S. A. Amitsur: Means of arithmetic functions.

This note contains simplifications of proofs and further applications of a symbolic method used for a unification of elementary proofs in number theory, which was introduced in Analyse de Math. vol. 5 (1956) pp. 273-314. Let g(n) be an arithmetic function and f(x) a function of a real variable. (Set $(I_g f)(x) = \sum_{n \leq x} g(n)n^{-1}f(x/n)$, then $I_g I_h = I_{g*h}$ where $(g*h)(n) = \sum_{d|n} g(d)h(n/d)$. Let $G(D) = \sum_{i=-p}^{\infty} g_i D^i$ be a formal power series in a symbol D which acts as the derivative d/d log x in the space of all polynomials in log x. Namely, $G(D)\log^n x = \sum_{i=-p}^{n} n!(n-i)!^{-1} g_i \log^{n-i} x$. Put $R_{g,n}(x) = [I_g - G(D)]\log^n x$, $R_{h,n}(x) = [I_h - H(D)]\log^n x$ and $R_{g*h,n}(x) = [I_g I_h - G(D)H(D)]\log^n x$. A relation is established between R_{g*h} and R_g and R_h. This is used to obtain by elementary methods various means (known and new) of arithmetic functions e.g. $\sum_{n \leq x} n^{-1}|\mu(n)|\log^k(x/n) = a_0(k+1)\log^{k+1} x + \ldots + a_k + O(x^{-1/3}\log x)$ with $a_0 = 6/\pi^2$. (Received December 4, 1958.)

553-106. P. M. Anselone: Ergodic theorems for discrete semi-Markov processes. Preliminary report.

Suppose that the transitions of a stationary Markov chain with a countable set of states occur at random positive integral times. Each transition to a state is followed by a wait in that state, the waits are independent random variables, and all the waits in a particular state have the same distribution. Let u_i be the reciprocal of the mean recurrence time of the ith state in the original Markov chain. Let e_i be the mean wait in the ith state. Under certain additional hypotheses, prob. (ith state at time t) $\rightarrow e_iu_i/\sum e_iu_i$ as $t \rightarrow \infty$. Various other theorems are obtained which reflect periodic behaviour of the stochastic process. In these cases the limit statement displayed above is correct if convergence is replaced by mean convergence. Similar, more general theorems hold if the wait in a particular state has a distribution which depends on that state and also on the following one. (Received December 4, 1958.)

553-107. F. G. Asenjo: On the formalization of the concept of internal relation.

Denote by σ(from Greek: $\sigma\tau\alpha s$), one primitive binary relation of presence at the same level as the primitive relation of membership ϵ. Let the terms of that relation σ be called societies and families of societies (or organisms), instead of elements and classes. Let that relation be internal in the sense that every society constitutes the remaining ones, but in a different rank. Hence, for one society s to be present in one family F, "$s\sigma F$", means that s has a multiple location in every other society of the family. Multiple locations are then organizing internal relations, and in the same way as one external relation R determines one class, so every multiple location L determines one family of societies which is univocal, and constitutes a whole of its own. Several axioms and metatheorems are introduced to formalize such a system of mathematical entities. (Received December 4, 1958.)

553-108. S. I. Askovitz: Some applications of centroids to problems in numerical analysis.

Elementary properties of centroids of finite sets of points have been employed to derive a number of rapid graphic techniques. Most of these methods require only pencil and straightedge, and may be of value in certain problems in numerical analysis. For example, it is possible, without any calculations,

to draw the straight line best fitting a series of points according to the least squares criterion. Other applications include the construction of a system of moving averages, the determination of the parameters of a frequency distribution, and some interesting results in finite differences and curve plotting. (Lantern slides available 4 × 3 1/4) (Received December 5, 1958.)

553-109. W. G. Bade and P. C. Curtis, Jr.: Continuity properties of homomorphisms of function algebras. Preliminary report.

Certain preliminary results are announced concerning the problem of the existence of a discontinuous homomorphism of C(S), the ring of all continuous functions on a compact Hausdorff space S, into an appropriate Banach algebra A. The following are two results which show that in some situations arbitrary homomorphisms have certain boundedness properties. Theorem: Let A be a Banach algebra with identity and let L be a family of commuting idempotents of A partially ordered in the usual way. Suppose L satisfies the following conditions (1) $1 \in L$: (2) $p,q \in L \rightarrow p \cdot q \in L$; (3) $p,q \in L$, $p \geq q$, $\rightarrow p - q \in L$. Let θ be a homomorphism of A into B(X), the ring of bounded operators on a Banach space. Then if L is a bounded set in A, $\theta(L)$ is a bounded set in B(X). Theorem: Let S be a totally disconnected compact Hausdorff space. Let θ be a homomorphism of C(S) into B(X). Then there exists a finite subset M of S such that θ is continuous on the ideal of all functions which vanish on some neighborhood of M. (Received December 4, 1958.)

553-110. G. E. Baxter: On the number of changes in sign.

Let $\{X_k\}$ be a sequence of independent, identically distributed random variables each with characteristic function $\phi(\xi) = E\{\exp[i\xi X_k]\}$, and let $S_0 = 0$, $S_n = X_1 + X_2 + \ldots + X_n$ $(n \geq 1)$. A change of sign occurs between S_k and S_{k+1} if $S_k > 0$ and $S_{k+1} \leqq 0$, or if $S_k \leqq 0$ and $S_{k+1} > 0$. Let $\widetilde{N}_n$ be the number of changes in sign so defined for the sequence of partial sums $S_0, S_1, \ldots, S_n$. Finally, denote by M the 2×2 matrix whose elements are $M_{11} = [(1 - t\phi)^2 - u^2 t^2 \phi^2]/(1 - t\phi)$, $M_{21} = -M_{12} = ut\phi/(1 - t\phi)$, and $M_{22} = 1/(1 - t\phi)$ where $|u| < 1$ and $|t| < 1$. There exists a unique matrix factorization of M in the form $(P_{ij})(Q_{ij})^{-1}$, where $P_{ij} = \delta_{i1} \cdot \delta_{j1} + \delta_{i2} \cdot \delta_{j2} + \int_{0+}^{\infty} e^{i\xi\alpha}\, dp_{ij}$ and $Q_{ij} = \int_{-\infty}^{0+} e^{i\xi\alpha}\, dq_{ij}$, and where P_{ij} and Q_{ij} are analytic in u and t. The matrix elements P_{ij} and Q_{ij} in these factors are connected with the number of changes in sign as defined above. In particular, $P_{21} = \sum_{n=1}^{\infty} t^n \sum_{m=0}^{n} u^m \int_{0+}^{\infty} e^{i\xi\alpha}\, d_\alpha P\{S_n < \alpha, \widetilde{N}_n = m\}$, and Q_{11}

$= \sum_{n=0}^{\infty} f^n \sum_{m=0}^{n} u^m \int_{-\infty}^{0+} e^{i\xi\alpha}\, d_\alpha P\{S_n < \alpha, \tilde{N}_n = m\}$. (Received December 4, 1958.)

553-111. W. E. Deskins: Some generalized Frattini subgroups of finite groups. Preliminary report.

Define $\phi_p(G)$ to be the intersection of all the maximal subgroups of the finite group G whose indices are not divisible by p, a prime divisor of the order of G, or G if no such subgroups exist. Then $\phi_p(G)$ has the following properties: (1) $\phi_p(G)$ is metanilpotent; (2) $\phi_p(H) \subseteq \phi_p(G)$ if H is a normal subgroup of G; (3) $\phi_p(G/H) = \phi_p(G)/H$ if H is a normal subgroup of G contained in $\phi_p(G)$. Now define $\phi_1(G)$ to be the minimal subgroup of G which contains all $\phi_p(G)$. Then $\phi_1(G)$ is a metanilpotent subgroup of G, and G is metanilpotent if and only if $G/\phi_1(G)$ is a nilpotent group. (A metanilpotent group is an extension of a nilpotent group by a nilpotent group.)(Received December 4, 1958.)

553-112. J. R. Büchi and C. C. Elgot: Decision problems of weak second order arithmetics and finite automata. Preliminary report. Part I.

1. Let L_1 be the class of formulas constructed out of atomic formulas $x_i \in F_j$, $y_k = x_\ell + 1$, $x_i < y_j$, by means of propositional connectives and quantifiers, $\exists x_i$, $\exists F_j$. The individual variables range over the natural numbers, N. Each formula $A[F_1,F_2,\ldots,F_r]$ in L_1 may be interpreted as representing a set (usually infinite) of finite sequences of r-tuples of zeros and ones as well as sets "isomorphic" to that set. Theorem. A set of finite sequences of objects drawn from a finite set is representable by a formula in L_1 iff it is representable by a finite automaton [cf. Kleene, Automata studies, or Copi, Elgot, and Wright, J. A. C. M., April, 1958]. 2. Corollary. The set of true sentences of L_1 is recursive. The corollary has been obtained by A. Ehrenfeucht and R. L. Vaught via Ehrenfeucht's theorem (unpublished) stating that the elementary theory of addition of ordinals is decidable. 3. Corollary. There are solution and synthesis algorithms relative to the class of all automata and L_1. [Cf. Büchi, Elgot, and Wright, these Notices, February, 1958, p. 98 for definitions.] 4. Each formula $A[F_1,F_2,\ldots,F_r]$ in L_1 defines $R \subseteq N^r$ via $F \leftrightarrow \sum_{n \in F} 2^n$. Corollary. The elementary theory of R, e.g., $x + y = z$ (Presburger), is decidable. (Received December 4, 1958.)

553-113. W. C. Fox: Differentiation on Riemann surfaces.

If F is a nonconstant meromorphic function on the Riemann surface W, and if u is a potential function on W with poles (including logarithmic ones) then there exists a meromorphic function G (constant if and only if u is a linear function of the real part of F) related to u and F as follows. To each point w there corresponds a local coordinate c mapping a disk centered at F(w) onto a neighborhood of w (with $c(F(w)) = w$) such that $(\partial uc(z)/\partial x) - i(\partial uc(z)/\partial y) = G(c(z))\,(F(c(z)))' = G(c)m(z - F(w))^{m-1}$ is valid for all $z = x + iy$ near F(w) unless w is a pole of F in which case the relation becomes $(\partial uc(1/t)/\partial x) - i(\partial uc(1/t)/\partial y) = G(c(1/t))(F(c(1/t)))' = G(c(1/t))mt^{-(m+1)}$ for all $t = x + iy$ near zero. In either case m is the multiplicity of F at w. Thus the ramification points, poles, and zeroes of G can be predicted by knowledge of u and F. If u is the real part of a meromorphic h then G is written $D_F(h)$. This "differentiation with respect to F" has all the usual properties of "differentiation with respect to z." Also $f = D_F(h)$ if and only if $\int f dF = 0$ around every closed path free of poles of f and F, and $D_F(G)D_G(F) = 1$. (Received December 4, 1958.)

553-114. P. C. Rosenbloom: Boundary value problems for hyperbolic equations.

It is shown that there are boundary value problems of the Dirichlet and Neumann types for equations such as $u_{xxyy} - \Delta u + u = f(x,y)$ in a rectangular region. The proofs are based on a priori estimates like those of Schauder. (Received December 5, 1958.)

553-115. Sigurdur Helgason: Wave equations on homogeneous spaces.

It is shown that the solutions of the wave equation $\partial^2 u/\partial x_1^2 - \partial^2 u/\partial x_2^2 - \ldots - \partial^2 u/\partial x_n^2 = 0$ can be characterized by the following mean value property: The Lorentzian mean value of u over the "sphere" S_r: $x_1^2 - x_2^2 - \ldots - x_n^2 = r^2$, is independent of r. By the Lorentzian mean value is meant the integral with respect to the invariant measure on the coset space H/K (which can be identified with S_r), where H is the linear group leaving the form $x_1^2 - x_2^2 - \ldots - x_n^2$ invariant and K is the subgroup of H that leaves the x_1-axis fixed. The signature +--...- is singled out among other indefinite metrics on R^n by the compactness of K. This is an exact analog of the classical mean value theorem for harmonic functions in R^n and the Darboux relation for spherical averages as well as Ásgeirsson's mean value theorem have counterparts in the Lorentzian metric.

However, due to the noncompactness of H, growth conditions at infinity must be imposed on the solution u. The proofs are group theoretic and generalize to certain other homogeneous spaces G/H having G-invariant metric of signature +--...- . (Received December 4, 1958.)

553-116. T. R. Jenkins and J. D. McKnight, Jr.: Ideals in rings of continuous functions.

For any ideal $I \subset C(X)$, where X is an arbitrary space, I^m denotes the m-closure of I and mI the ideal consisting of all $f \in C(X)$ for which there exist $h \in C(X)$ and $g \in I$ such that $Z(f) \supset X - Z(h) \supset Z(g)$, where Z(f), Z(g), Z(h) are the sets of zeros of f,g,h, respectively. It has been shown (Gillman, Henriksen and Jerison, 1954) that $mI \subset I$ and $(mI)^m = I^m$. This result is augmented nontrivially as follows: If J is an ideal of C(X) and $J \subset I^m$, then $mJ \subset mI$. Equivalently, if $J^m = I^m$, then $mI \subset J$. It follows easily that $I^m = J^m$ iff $mI = mJ$, that $mmI = mI$ for each ideal I, and that $mI = \bigcap \{J: J \text{ is an ideal of } C(X) \text{ and } J \subset I^m\}$. The ideals of C(X) are thereby seen to be contained in equivalence classes such that each class has extremal elements of the form mI and I^m, where I is any element of the class. This constitutes an extension of well-known unpublished facts concerning C(X) for a compact Hausdorff space X. (Received December 4, 1958.)

553-117. R. B. Kelman: On the asymptotic behaviour of a differential-functional equation.

Consider $y'(x) = g(x)\cos(ux + vy(px + q) + w)$(*) where u, v, w, p and q are real constants such that $u \neq 0$, $v \neq 0$, and $p > 1$, or $p \geq 1$ and $q \geq 0$. Let $X > 0$ be a constant and assume g(x) is a function of bounded variation in (X,∞) such that $g(x) \to 0$ as $x \to \infty$ and the total variation of g(x) in (x,∞) is $Ox^{-\delta}$ for some fixed $\delta > 0$. The following results are obtained. If p is not the reciprocal of an algebraic integer, then each real solution y(x) of (*) satisfies an order relation $y(x) = K + Ox^{-\delta}$ where K is an arbitrary constant. For each polynomial $C = Cnp^n + \ldots + C_0$ and $Cn \neq 0$, $C_0 \neq 0$, with rational integral coefficients, define $\Gamma(C)$ to be the sum $\delta \sum_{j=1}^{n} |C_j|$ augmented by 2δ for each j, $1 \leq j \leq n - 1$, such that $Cj = 0$. For given p, let $\Gamma' = \inf\{\Gamma(C): C(p) = 0\}$. Now assume that p is the reciprocal of an algebraic integer. Then each solution satisfies $y(x) = K + O(x^{1-\Gamma'} + x^{-\delta})$. This result is best possible in the sense that Γ' cannot be replaced by a larger number. The proof of these results proceeds by itera-

tion and is facilitated by the use of an identity (too complicated to be given here) due to van der Corput. (Received December 4, 1958.)

553-118. G. F. Leger and Shigeaki Tôgô: On characteristically nilpotent Lie algebras.

Let L be a lie algebra with derivation algebra D(L). Let $L^{[1]} = D(L)L = \{\sum D_i x_i | x_i \in L, D_i \in D(L)\}$ and define $L^{[k+1]} = D(L)L^{[k]}$ inductively. Dixmier and Lister, Proc. Amer. Math. Soc. (1957), have called a Lie algebra characteristically nilpotent if $\exists$ an integer $k \ni L^{[k]} = 0$. Theorem 1. L is char. nilp. $\rightleftarrows$ D(L) is nilpotent and L is not one dimensional. Theorem 2. Let the ground field of L be alg. closed of char. 0. Then L is char. nilp. $\rightleftarrows$ all semi-simple automorphisms of L are of finite order. Theorem 3. Let $L = L_1 + \ldots + L_n$, direct sum of ideals. Then, L is char. nilp. $\rightleftarrows$ each L_i is char. nilp. Theorem 4. Let L be char. nilp. If D(L) annihilates the center of L then L is not a derived algebra. Theorem 5. Let L be char. nilp. and let m and n be the smallest integers such that $L^m = 0$ and $L^{[n]} = 0$. If $m - 1 > [(n + 1)/2]$, then L is not a derived algebra. (Received December 4, 1958.).

553-119. Walter Littman: A strong maximum principle for weakly L-subharmonic functions.

Let D be a domain in n-dimensional Euclidean space, and let $L = \sum_{i,j} A_{ij}(x)\partial^2/\partial x_i \partial x_j + \sum_i A_i(x)\partial/\partial x_i$ be an elliptic operator defined on $C^2(D)$. (Ellipticity: $\sum A_{ij}\xi_i\xi_j$ is positive definite). Furthermore suppose that $A_{ij} \in C^2(D)$ and $A_i \in C^1(D)$, and denote by L^* the formal adjoint of L. If u is measurable and essentially bounded from above in D, and if $\int_D u(x)\, L^*v(x)dx \geq 0$ for all non-negative v in $C^2(D)$ with compact support in D, we say u is weakly L-subharmonic in D. Theorem: If u is weakly subharmonic in D and assumes its essential supremum M at a point of continuity of u (in D) then $u = M$ almost everywhere in D. (Received December 4, 1958.)

553-120. H. L. Loeb: Algorithm for Chebycheff approximations using the ratio of linear forms. Preliminary report.

Given: $a^i \in E_n$ $(i = 1,\ldots,v)$, $b^i \in E_m$ $(i = 1,\ldots,v)$, and real numbers $B_1,\ldots, B_v, \delta_1, \delta_2$ where $0 < \delta_1 < \delta_2 < 1$. Define $S = \{x \in E_{n+m}: \delta_1 \leq \sum_{\ell=1}^m x_{n+\ell} b_\ell^i \leq \delta_2;\ i = 1,\ldots,v\}$. Problem: Find M such that $M = \min_{x\in S} \max_{1\leq i\leq v} |B_i - (\sum_{j=1}^n x_j a_j^i) \div (\sum_{\ell=1}^m x_{n+\ell} b_\ell^i)|$. Let $-a^{i+v} = a^i, -B_{i+v} = B_i$, and $b^{i+v} = b^i$

$(i = 1,\ldots,v)$. Algorithm: A sequence $\{M_r\}$ $(r = 1,\ldots)$ is generated by the following relationship, $M_r = M_{r-1} + \min_{x\in S}\max_{1\le i\le 2v}\{-\sum_{j=1}^{n} x_j a_j^i + (B_i - M_{r-1})\sum_{\ell=1}^{m} x_{n+\ell} b_\ell^i\}$ $(r = 1,\ldots)$ where $M_0 = 0$. It can be shown that $\lim_r M_r = M$. Linear programming techniques can be used to find each M_r. In writing this paper the author was influenced by some remarks in an article of L. S. Shapley's. (Received December 4, 1958.)

553-121. R. W. McKelvey: Singular second order differential boundary problems of Fourier type.

Let $g(t)$ be real, continuous, and absolutely integrable on $0 \le t < \infty$, and consider the differential boundary problem $u'' + [2 - g(t)]u = 0$, $0 \le t < \infty$, $\sin\alpha\cdot u(0) - \cos\alpha\cdot u'(0) = 0$. Various methods have established the related expansion theorem for a function $f(t)$ of class $L_2(0,\infty)$, convergence being in the sense of L_2 norm. E. C. Titchmarsh [Eigenfunction expansions, Oxford, 1946] has shown that if $f(t)$ in addition belongs to class $L(0,\infty)$ and is b.v. near a point t_1, then the expansion converges in the ordinary sense at the point t_1 to $[f(t_1 +) + f(t_1 -)]/2$. The method of the present paper avoids entirely L_2 assumptions and methods, and relies instead upon contour integrals obtained by a limiting process from a finite interval problem. Much of the work is in obtaining suitable asymptotic estimates of the solutions of the differential equation, and in making these appraisals it is imposed that $\int_1^\infty t|g(t)|dt < \infty$. Then for any function $f(t)$ of class $L(0,\infty)$ the expansion is proved to be equiconvergent with the Fourier cosine integral or, (when $\alpha = \pi/2$), with the Fourier sine integral. (Received December 4, 1958.)

553-122. R. D. McWilliams: Projections of separable spaces of bounded sequences onto the space of convergent sequences.

Let (m) be the Banach space of bounded sequences of real numbers, and let (c) and (c_0) be the subspaces of (m) consisting of all convergent sequences and all sequences converging to zero respectively. Sobczyk [Bull. Amer. Math. Soc. vol. 47 (1941) pp. 938-947] has shown that if B is a separable subspace of (m) containing (c_0), then there exists a projection of norm 2 of B onto (c_0). Using Sobczyk's proof, we show that if B is a separable subspace of (m) properly containing (c), then there exists a projection of norm not more than 3 of B onto (c), and every projection of B onto (c) has norm at least 2. A separable subspace B is exhibited such that every projection of B onto (c) has norm at

least 3. On the other hand there exist subspaces containing denumerably many linearly independent elements not in (c), which have projections of norm 2 onto (c). (Received December 4, 1958.)

553-123. Leo Moser: On the minimal overlap problem of P. Erdős.

Let $\mathcal{E}$ be a separation of the integers $1, 2, \ldots, 2n$ into two classes $\{a_i\}$ and $\{b_i\}$ with n elements in each class. Let M_k denote the number of solutions of $a_i - b_j = k$ and let $M = M(n) = \min_{\mathcal{E}} \max_k M_k$. It is shown that for n sufficiently large $.389 > M/n > .357$. This improves at least for special cases, results of P. Erdős (Riveon Lematematika vol. 9 (1955) p. 48), P. Scherk, T. S. Motzkin, K. E. Ralston, J. L. Selfridge, L. C. Hsu, S. Swierczkowski, J. Mycielski, F. Kasch and H. Schell. (Received December 4, 1958.)

553-124. J. C. C. Nitsche: On Riemann's problem of function theory.

In §19 of his dissertation (1851) B. Riemann formulated the problem: To determine an analytic function $w = u + iv$ in a domain T satisfying the boundary condition $B(u,v;s) = 0$ on ∂T. The problem has never been attacked in this generality. D. Hilbert (1904) and F. Noether (1921) gave the solution for the case $B \equiv a(s)u + b(s)v + c(s)$. Consequently much research was done and generalizations to linear and nonlinear systems, multiply connected domains, etc. were proved. (For literature see [1] N. I. Muskhelishvili, Singular integr. equ.; [2] Joh. Nitsche, Math. Nachr. vol. 14 (1955); [3] I. N. Vekua, Mat. Sb. vol. 31 (73) (1952).) Author considers nonlinear boundary value problems $w_{\bar{z}} + A(z)w + A'(z)\overline{w} = 0$, $B(u,v;s) = f(s)$ in a simply connected domain. An argument similar to one of Joach. Nitsche (M. Z. vol. 60 (1954)) leads one to assume that B be harmonic in u and v, e.g. $B(u,v;s) = \mathrm{Re}[\sum_{j=1}^{k}(a_k(s) - ib_k(s))w^k]$. Here $a_k^2 + b_k^2 \neq 0$, $a_1^2 + b_1^2 \neq 0$. The indices $n_j = [\arg(a_j + ib_j)]_{\partial T}$ are important. Existence theorems are proved in certain cases, for instance $f = 0$, $n_1 \geq 0$ and $A = B = 0$, $n_k \geq 0$. The proofs use the representation formulae of [2]. (Received December 4, 1958.)

553-125. Emanuel Parzen: Statistical inference on time series by Hilbert space methods. I.

This paper attempts to show how Hilbert space methods (which were introduced in the 1940's to clarify the probabilistic structure of time series) can be used to clarify, and to solve, various problems of statistical inference

on time series. We show the importance of the theorem proved by Loève (in 1948, in the appendix to P. Lévy, Processus stochastiques et mouvement Brownien) that every random function of second order may be represented by a reproducing kernel Hilbert space. It is shown how the problems of minimum variance unbiased estimation, and minimum variance unbiased estimation and prediction may be reduced to the solution of the following generalized integral equation; given an abstract Hilbert space H, a family of vectors $\{k(t), t \in T\}$ in H, and a function on the index set T, to find the vector x in H of minimum norm satisfying $(x,k(t)) = f(t)$, $t \in T$. This problem may be solved in terms of the reporducing kernel Hilbert space corresponding to the kernel $K(s,t) = (k(s), k(t))$. (Received December 4, 1958.)

553-126. Barth Pollak: Quadratic forms over rings.

Witt proved that congruence of n-ary quadratic forms over any field k of characteristic $\neq 2$ is characterized by determinant mod squares and a certain quaternion algebra over k for $n = 1,2,3$. In this paper we partially extend Witt's result to integral domains D of characteristic $\neq 2$. More precisely, with every nonsingular 3×3 symmetric matrix f with elements in D is associated a certain 4-dimensional algebra Q(f) over D. (Q(f) was first introduced by Linnik and Pall.) We then prove the following Theorem: Let f and g be nonsingular 3×3 symmetric matrices over D. Then f and g are congruent over D if and only if: (1) $\det f/\det g = e^2$ for some unit e in D, (2) Q(f) and Q(g) are D-isomorphic. This extends Witt's result for $n = 3$. Of course $n = 1$ is trivial. If D is such that Witt's cancellation theorem holds, the above theorem will take care of $n = 2$. For general D, however, $n = 2$ eludes us. (Received December 4, 1958.)

553-127. Louis Solomon: The representation of finite groups in algebraic number fields.

Let G be a finite group of order g, let χ be an absolutely irreducible character of G, and let K be a field of characteristic zero. Previous work of R. Brauer has reduced the determination of the Schur index $m_K(\chi)$ of χ over K to determination of the Schur indices of characters of certain solvable subgroups of G. The present paper constructs splitting fields for these characters. The splitting fields, together with known results, show that $m_K(\chi)$ divides $2\prod(q_i - 1)$ where the product is over all primes q which divide g. Thus if G is a p-group, p an odd prime, it follows that $m_K(\chi) = 1$. Hence every absolutely

irreducible representation of a p-group may be written in the field of its character. There is a similar statement for p = 2. The results also yield the known fact that any representation of G may be written in the field of g-th roots of unity. (Received December 4, 1958.)

553-128. W. F. Stoll: Meromorphic modifications.

Let G and H be pure n-dimensional complex spaces, $A \subset G$. Let $M = G - A$ and $N = H - B$ be thin and closed. A holomorphic map τ: $A \to B$ defines a modification μ if τ is one-to-one, $\tau(A) = B$ and if τ^{-1} is holomorphic. The map τ^{-1}: $A \to B$ defines the inverse modification μ^{-1}. The modification μ is said to be R-meromorphic if $\overline{\{(P,\tau(P))|P \in A\}}$ is an analytic subset of $G \times H$. Denote an irreducible, locally irreducible, one-dimensional analytic subset L of an open subset of G a complex curve. The modification μ is said to be weakly meromorphic (resp. meromorphic, resp. gapfree) if the set $\sum(P,L) = \{Q|\lim_{\nu\to\infty}(P^\nu, \tau(P^\nu)) = (P,Q)$ and $P^\nu \in L \cap A\}$ consists of at most (resp. exactly, resp. at least) one point for every $P \in M$ and every complex curve L of G when $L \cap M = \bar{L} \cap M = \{P\}$. A modification μ has a certain property in both directions if this property holds for both μ and μ^{-1}. If $n = 2$, then a modification is weakly meromorphic if and only if it is R-meromorphic. If $n = 3$, then a modification is meromorphic in both directions if and only if it is R-meromorphic and gapfree in both directions. (Received December 4, 1958.)

553-129. D. W. Wall: Basic algebras of UMFR algebras.

For $\mathcal{A}$ a finite dimensional algebra with unit element over a field, let $\mathcal{A}^*$ be the basic algebra of $\mathcal{A}$. The classification of the subclasses of the UMFR algebras (Algebras with unique minimal faithful representations) is the same as that of an earlier paper [Duke Math. J. vol. 25 (1958) pp. 321-329]. The following relationships between $\mathcal{A}$ and $\mathcal{A}^*$ are obtained: $\mathcal{A}$ is type UMFR, B, AC, or ABC, if and only if $\mathcal{A}^*$ is type UMFR, B, AC, or ABC, respectively; if $\mathcal{A}$ is type A or AB, then $\mathcal{A}^*$ is type UMFR or type B, respectively; if $\mathcal{A}^*$ is type A or AB, then $\mathcal{A}$ is type AC or ABC, respectively; if $\mathcal{A}^*$ is type A or AB, then $\mathcal{A}^*$ is type AC or ABC, respectively. These results are obtained by using: (1) the characterizations of the subclasses in terms of the socles of the primitive ideals of $\mathcal{A}$ and $\mathcal{A}^*$ [Theorem 5, loc. cit.]; (2) the decompositions of $\mathcal{A}$ and $\mathcal{A}^*$ into direct sums of primitive ideals; (3) the decompositions of the socles of the primitive ideals of $\mathcal{A}$ and $\mathcal{A}^*$. (Received December 4, 1958.)

553-130. G. S. Young: A reflection theorem for light interior functions.

A theorem somewhat more general than the following is proved: Let f and g be sense-preserving functions, f being light and interior in the open upper half plane and continuous on the x-axis, and g being light and interior in the open lower half plane and continuous on the x-axis. Suppose that $f(x,0) = g(x,0)$ for all x, and that f(x,0) is always on the x-axis. Then the function ϕ defined as f in the upper plane and g in the lower plane differs from a light interior function only in that there are intervals of constancy of ϕ on the x-axis, which can be removed by a monotone map. The result casts some light on existence theorems for partial differential equations, and has applications in cluster set theory. (Received December 4, 1958.)

553-131. W. A. Beyer: Rademacher series with geometric coefficients. Preliminary report.

Let $B(a,r) = \{x \mid \sum_{i=1}^{\infty} r^i R_i(x) = a;\ 0 < x \leq 1\}$ where a is a real number, $0 < r < 1$, and $R_i(x)$ is the ith Rademacher function. It is known that if $0 < r < 1/2$, B(a,r) is empty unless a belongs to a certain perfect set P(r) of measure zero. If $a \in P(r)$, then B(a,r) is one point. The purpose of this paper is to study B(a,r) for $1/2 < r < 1$. $\dim_H$ denotes Hausdorff dimension. We assume that $|a| < r/(1 - r)$. It is shown that if $((5 - 4/n)^{1/2} - 1)/2(1 - 1/n) < r < 1$ $(n > 1)$, then $1 - |\log_2 r| \geq \dim_H B(a,r) \geq 1/n$ except that the left inequality may not hold for a set of a of Lebesgue measure zero. The left inequality is due to John R. Kinney. The inequalities are probably not the best possible. Now suppose $r = ((5)^{1/2} - 1)/2$. Then B(a,r) is countably infinite if there exist n and x such that $a = \sum_{i=1}^{n} r^i R_i(x)$; otherwise B(a,r) is uncountably infinite. If $1/2 < r < ((5)^{1/2} - 1)/2$, there exist a's for which B(a,r) is single point. The proof uses a well known measure preserving mapping of the unit interval into an n-dimensional cube. (Received December 4, 1958.)

553-132. Pinchas Mendelson: On dynamical systems without improper saddle points. I.

Let R be a locally compact, separable metric space and let D = (F(p,t); R) be a dynamical system defined in R. We assume that D has no improper saddle points. Theorem 1. A necessary and sufficient condition for a motion f(p,t) to be positively (negatively) stable according to Lagrange is that its ω-limit set (α-limit set) be nonempty. Corollary. Every motion positively (negatively)

stable in the sense of Poisson is positively (negatively) stable in the sense of Lagrange. In particular, every recurrent motion is Lagrange stable. Theorem 2. Every point which is Lagrange unstable is wandering. Theorem 2 implies, but is not implied by the known result that every unstable dynamical system without an improper saddle point is completely unstable. (Received December 4, 1958.)

553-133. R. D. Anderson: Minimal sets under flows on tori and derived spaces.

The author gives a rather general constructive device for exhibiting certain subsets of tori as (unique) minimal sets under discrete (or, in some cases, continuous) flows. A corollary of the technique is the theorem that there exist a homeomorphism f of the 3-dimensional torus T and a subcontinuum M of T such that M is the universal curve and for any point p of T, $M \subset Cl\{f^i(p)\}$ and $Cl\{f^i(p)\} - M$ is either null or a countable set of isolated points. It follows that the universal curve is a minimal orbit closure. The paper contains contributions to several problems stated by Gottschalk in Minimal sets: An introduction to topological dynamics (to appear in Bulletin Amer. Math. Soc.). (Received December 5, 1958.)

553-134. W. G. Bade: A class of non-self-adjoint boundary conditions for the Laplacian. Preliminary report.

Let G be a bounded domain in the plane with $C^{1,1}$ boundary. We denote by $D_1(G)$ the class of functions u in $L_2(G)$ with two strong derivatives locally in G such that the values of u and $\partial u/\partial n$ on the parallel contours ∂G_r converge to limits $\tilde{u}$ and $\tilde{u}_n$ in $L_2(\partial G)$ as $r \to 0$. (Here ∂G_r is the set of points in G whose distance from ∂G equals r.) Let L be an arbitrary bounded operator in $L_2(\partial G)$ and let T_L denote the negative Laplacian on the domain $D(T_L) = \{u | u \in D_1(G), \tilde{u}_n - L\tilde{u} = 0\}$. Then T_L is closed, densely defined, and has compact resolvent in $L_2(G)$. Moreover, $(T_L)^* = T_{L^*}$. The spectrum of T_L lies in a sector of opening less than π containing the positive real axis. These theorems extend results of R. S. Freeman (Abstract 549-37) for the case $L = L^*$. (Received December 5, 1958.)

553-135. Hsin Chu: Bonded groups.

Let G be a topological group. A subgroup H of G is called syndetic if there exists a compact subset K of G such that $H \cdot K = G$. A topological group is called bonded if every nontrivial cyclic subgroup is syndetic. We study the structure of locally compact bonded groups. We prove that every locally compact bonded group is either (a) compact, or (b) the group of all integers with the discrete topology, or (c) the group of all real numbers with the usual topology. In this paper, nonlocally compact bonded groups are also studied. (Received December 5, 1958.)

553-136. V. F. Cowling and W. C. Royster: On the Euler and Taylor summation of Dirichlet and Taylor series.

The effectiveness of the Euler-Knopp $\mathfrak{E}(r)$ and the Taylor V_α series to series transformation methods are studied from the following standpoint. Consider a particular value of the parameters, r and α. Let $b_n = b_n(z_0) = \sum a_{nk}a_n(z_0)$ be the transform with fixed parameter value of the series $\sum a_n(z)$ at $z = z_0$. Suppose $g(u) = \sum b_n(z_0)u^n$ has certain function theoretic properties then the effectiveness of the methods is determined in terms of the fixed parameter value and z_0. It is shown among other things that if $0 < r_0 < 1$ and $g(u) = \sum b_n(r_0,z_0)u^n$ converges in the unit circle and is regular at $u = 1$, where $b_n(r_0,z_0)$ is the $\mathfrak{E}(r_0)$ transform of $\sum a_n(n+1)^{-z}$ at $z = z_0$, then for each $r < r_0$ the series $\sum a_n(n+1)^{-z}$ is $\mathfrak{E}(r)$ summable for all finite z. By considering other function theoretic properties which g(u) may satisfy results of a similar nature are obtained for the $\mathfrak{E}(r)$ and V_α methods. (Received December 5, 1958.)

553-137. Chandler Davis: On functional calculus.

Let $\mathcal{H}$ be a hilbert space, $\mathcal{H}^k$ the hilbert space of k-tensors. For A hermitian on $\mathcal{H}$, and f a suitable real function of k real variables, f(A) is defined as a hermitian operator on $\mathcal{H}^k$, in analogy to the convention for $k = 1$. In particular, if f is a sufficiently differentiable function of 1 variable, its kth divided difference is a function of $k + 1$ variables, giving operator $f^{(k)}(A)$ on $\mathcal{H}^{k+1}$. Let ϕ_k be the contraction function from operators on $\mathcal{H}^k$ to operators on $\mathcal{H}$, determined by $\phi_k(F \otimes \dots \otimes G) = F \dots G$. The main subject of the present paper is the "Taylor formula" $Bf(A + tB) = \sum_k t^k \phi_{k+1}((B \otimes \dots \otimes B)f^{(k)}(A))$. (Received December 5, 1958.)

553-138. J. W. Addison: Some consequences of the axiom of constructibility.

Let N be the set of natural numbers, let lower case Roman (Greek, German) letters range over N (over N^N, over a product of copies of N and N^N), and if $C \subseteq N^N$ let $\{\sum^1_k[C], \prod^1_k[C]\}_k$ be the hierarchy of predicates, functions, and sets analytical in functions in C. A full proof is given that the axiom of constructibility (A) implies the following: there exists an Ω_1 well-ordering $\prec$ of N^N such that for any subset C of N^N and any predicate R recursive in functions in C the set $\hat{\alpha}\hat{\beta}(E\beta_1)[\beta_1 \prec \beta \ \& \ (E\alpha)(x)R(\alpha, \beta_1, \alpha, x)]$ is in $\sum^1_2[C] \cap \prod^1_2[C]$. Using this result a "best-possible" table showing the power of the least function operator on the classes of the analytical hierarchy is computed. The table is applied to obtain several results of which the following are samples: (1) (A) implies that the class of $\sum^1_{k+1}[C] \cap \prod^1_{k+1}[C]$ functions is a basis for $\prod^1_k[C]$ if $k > 0$; (2) (A) implies that all $\sum^1_2[\emptyset] \cap \prod^1_2[\emptyset]$ -degrees are comparable. Applying lemmas used in the proof of the main theorem it is also shown that the class of constructible elements of N^N is a $\sum^1_2[\emptyset]$ class. (Received December 5, 1958.)

553-139. R. W. Bass: The periodic solutions of the n-body problem.

Let real $V(x) \equiv V(-x) \geqq 0$ be defined, $\in C^{(1)}$, on $E^m - \Sigma, \Sigma a$ closed nowhere dense set, and let $V(\alpha x) \equiv \alpha^d V(x)$, $\alpha \geqq 0$. ($\mathscr{A}$) denotes system $\ddot{x} = -\operatorname{sgn}(d)\operatorname{grad} V(x)$, $(\cdot = d/dt)$. A periodic solution $x(\nu t)$ is symmetric, of frequency $\nu > 0$ if $x(\theta + \pi) \equiv -x(\theta)$, $\theta \in [0, \pi]$. Clearly $1/2\nu^2 \left|dx/d\theta\right|^2 + \operatorname{sgn}(d)V(x(\theta)) \equiv h = \text{const.}$, $\theta \in [0, 2\pi]$. Call $x(\theta)$ a normalized symmetric periodic solution (n.s.p.s.) if also $\int_0^{2\pi} \left|dx/d\theta\right|^2 d\theta = \pi$; then $\nu = 2(|h/(1 + 2/d)|)^{1/2} > 0$, $\|x(\theta)\| \leqq \pi/2(2)^{1/2}$. Theorem 1. A n.a.s.c. that ($\mathscr{A}$) have at least one n.s.p.s. is that (i) $d \neq -2, \neq 0, \neq 1$; (ii) $\operatorname{sgn}(h) = \operatorname{sgn}(1 + 2/d)$. Theorem 2. Assume (i). Let $S^\infty \equiv \{u(\theta) | u(\theta + \pi) \equiv -u(\theta) \in E^m, \theta \in [0, \pi], \int_0^\pi \left|u(\theta)\right|^2 d\theta = \pi/2\}$. Let $\bar{V}\{u(\theta)\} = (1/\pi)\int_0^{2\pi} V(2^{-1}\int_0^\pi K(\theta,\sigma)u(\sigma)d\sigma)d\theta$, where $K(\theta,\sigma) = +1$, $0 \leqq \sigma \leqq \theta \leqq \pi$; $= -1$, $0 \leqq \theta < \sigma \leqq \pi$; $\equiv -K(\theta + \pi, \sigma)$, $\theta \in [0, \pi]$. On S^∞, $\bar{V}$ is a "mildly singular" completely continuous scalar whose critical points $u(\theta)$ correspond precisely to the n.s.p.s.'s $x(\theta)$ of ($\mathscr{A}$) by quadratures: $x(\theta) \equiv 2^{-1}\int_0^\pi u(\sigma)d\sigma + \int_0^\theta u(\sigma)d\sigma$, $\equiv -x(\theta + \pi)$, $\theta \in [0, \pi]$; $\nu^2 = |d|\bar{V}\{u(\theta)\}$. Theorem 3. On S^∞, $\bar{V}$ has at least one critical point; if ($\mathscr{A}$) is the Newtonian n-body problem, then $\bar{V}$ has no more than n "topologically necessary" critical points. N. B. Theorem 2 yields a numerical algorithm for finding the Fourier coefficients of all n.s.p.s.'s of ($\mathscr{A}$). (Received December 5, 1958.)

553-140. C. C. Faith: Concerning noninvariant submodules of simple matrix and local matrix rings. II.

A ring A is radical over a subring B in case some power of each $a \in A$ lies in B. In "A note an algebraic division ring extensions" (announced at Edinburgh, and forthcoming in Proc. Amer. Math. Soc.), the following theorem was established: (1) If A is a simple ring having minimal one-sided ideals, and if A is radical over a proper subring B, then A is a (commutative) field. The following related results are now established. (2) If A is a simple algebra with an idempotent $\neq 0, 1$ over a field $\neq GF(2)$, then A cannot be radical over any proper subalgebra. (3) If A is a complete ring of $n \times n$ matrices over a ring, $n > 1$, then A cannot be radical over any proper subring. (4) If A is a simple algebraic algebra over a field $\neq GF(2)$, and if A is radical over a proper subalgebra B, then either A is a field, or else A is a nil algebra. (1) is used to obtain the division ring case of (4). (2) is a consequence of a result stating that A is generated (as an algebra) by the conjugates of any idempotent $\neq 0,1$. This, and other results on the generation of A as defined in (2), e.g., each $a \in A$ is a sum of quasi-regular elements, can be obtained as an application of a theorem of Amitsur (See (I) of the present title, these Notices June 1958, Abstract 546-69; this paper is forthcoming in Proc. Amer. Math. Soc.). (Received December 5, 1958.)

553-141. P. C. Hammer: Generalized continuity.

In this paper we examine in detail the consequences of the definitions of continuities depending on expansive functions which generalize the usual closure functions. An expansive function g relative to a set M maps the class of all subsets of M into that class and is such that $gX \supset X$ and if $X \supset Y$ then $gX \supset gY$. These functions are described in our paper "General topology, symmetry, and convexity," Transactions of the Wisconsin Academy of Sciences, Arts and Letters, vol. 43, 1955. If t is a transformation from M onto a set M_1 then t is (u,v)-continuous provided $t(uX) \subset v(tX)$ for all $X \subset M$ where u and v are expansive functions in M and M_1 respectively. General properties of the class of all pairs $(u_\alpha v_\alpha)$ such that a transformation t is (u_α, v_α)-continuous are given. (Received December 5, 1958.)

553-142. Guy Johnson: Limit-convergence of function sequences.

Let $\{f_n(z)\}$ be a sequence of complex valued functions defined on a set A in the compact z-plane. Let A' denote the set of limit points of A. Suppose $z_0 \in A'\{z_n\} \subset A$, and $z_n \to z_0$. Denote $\rho_n = \sup_p |f_n(z_n) - f_{n+p}(z_n)|$. The sequence $\{f_n(z)\}$ is said to be limit-convergent at z_0 relative to A if and only if $\rho_n \to 0$ for each sequence $\{z_n\}$. These two theorems follow. $\{f_n(z)\}$ converges uniformly on A $\Leftrightarrow \{f_n(z)\}$ is limit-convergent relative to A at each point of A'. The next theorem relates limit-convergence to Caratheodory's continuous convergence. Suppose $\{f_n(z)\}$ converges to f(z) on A, $z_0 \in A'$ and the sequence is limit-convergent at z_0 relative to A. Then $\{f_n(z)\}$ is continuously convergent at z_0 relative to A $\Leftrightarrow$ f(z) is (or may be extended to a function) continuous at z_0 relative to A. (Received December 5, 1958.)

553-143. W. B. Jurkat: Semi-groups of Toeplitz matrices.

This paper contains a characterization (in terms of the infinitesimal behavior) of matrix-functions $P(t) = (p_{jk}(t))$ satisfying $p_{jk}(t) \geq 0$, $\sum_n p_{jn}(t) = 1$, $\sum_n p_{jn}(t)p_{nk}(s) = p_{jk}(t+s)$, $\lim_j p_{jk}(t) = 0$, $p_{jk}(+0) = \delta_{jk} = p_{jk}(0)$ for all $j,k \geq 0$ and $s,t > 0$. Thus P(t) defines a totally regular summability method of "order t" or a regular Markov process with nullsequences in each column. Using ideas of the theory of semi-groups of positive matrices, in particular of Markov processes, e.g. the following is proved: $p'_{jk}(0) = q_{jk}$ exists, both Kolmogorov differential equations hold, P(t) is Feller's minimal process associated with $Q = (q_{jk})$, conversely Q with certain characterizing properties generates P(t), also P(t) defines a strongly continuous contraction semi-group of linear and bounded operators on the space of nullsequences, and the infinitesimal generator is given in a natural way by Q. An example is provided by the theory of Hausdorff methods. (Received December 5, 1958.)

553-144. L. H. Lange: A non-Euclidean analogue to a theorem of H. Milloux and its relationship to a theorem of W. Seidel.

Let f(z) be an unbounded holomorphic function in $|z| < 1$. Let S be a spiral on which f(z) remains bounded. For a large sub-class of such functions W. Seidel has proved, with an argument involving normal families, the existence of a spiral S' having the property that in the union of all non-Euclidean (abbreviated n-E) discs having their n-E centers on S' and having an arbitrary fixed positive n-E radius the function f(z) assumes infinitely often every finite complex value,

with at most one exception (Nagoya Math. J., December, 1958). As he points out, this is an analogue to a classical theorem of G. Julia. The present author, after developing appropriate forms of Schottky's theorem, proves the existence of "a set of ρ-discs for f", a sequence of n-E discs whose n-E radii tend to zero and in whose union the function assumes infinitely often every finite complex value, with the exception of at most one value. These ρ-discs are analogous to the classical cercles de remplissage of H. Milloux. After demonstrating the existence of a spiral S'' on which the n-E centers of a set of ρ-discs lie and on which $|f(z)| \to \infty$, the author points out the relationship of the present results to the Julia-Seidel theorem. (Received December 5, 1958.)

553-145. R. C. Lyndon: Direct products of algebras.

For terminology, see Morel-Scott-Tarski (Abstract 550-9) and Chang-Morel (J. Symbolic Logic, vol. 23). I. Every existential sentence is easily seen to be equivalent to a disjunction of existential Horn sentences. Theorem. Every existential sentence preserved under direct products is equivalent to a single existential Horn sentence. II. If $J_0,\dots,J_N$ are ideals in the boolean algebra 2^I of all subsets of a set I, and $2^I/J_0$ is isomorphic to the direct sum of the $2^I/J_n$ $(n = 1,2,\dots,N)$, then a reduced product, $\mathcal{A}^I/J_0$, is isomorphic to the direct product of the $\mathcal{A}^I/J_n$. If $K = K_1 \cup \dots \cup K_N$, each K_n an elementary class, a reduced product $\mathcal{A}$ of algebras from K is a direct product of algebras $\mathcal{A}_n$, where each $\mathcal{A}_n$ is a reduced product of algebras from K_n. Theorem. If a sentence of the monadic calculus without identity is preserved under direct products, then it is preserved under reduced products. (Received December 5, 1958.)

553-146. Albert Nijenhuis: Hyperconvexity in Riemannian manifolds.

A convex open set V in a Riemannian manifold is defined to be γ-hyperconvex $(0 < \gamma \leqq 1)$ if the following holds: For every $\epsilon > 0$ and every two pairs of points y_1,z_1; y_2,z_2 such that $\rho(y_1,y_2) < \epsilon$, $\rho(z_1,z_2) < \gamma\epsilon$ the geodesic segments y_1z_1, y_2z_2 have the property that corresponding points have distance less than ϵ. Here, $\rho(y,z)$ denotes the distance; and the points x_1,x_2 of y_1z_1, y_2z_2 respectively are corresponding ones if $\rho(x_1,y_1) \div \rho(y_1,z_1) = \rho(x_2,y_2) \div \rho(y_2,z_2)$. In spaces of negative curvature, all convex sets are 1-hyperconvex, as a consequence of much stronger results of H. Busemann [Geometry of geodesics, Theorems (36.4) and (36.17)]. In other cases we have the result that $\gamma = \cos Dk^{1/2}$, where D is the diameter of V, and k the supremum of the sectional curvature in V; provided $kD^2 < \pi^2/4$. (Received December 5, 1958.)

553-147. Steven Orey: Recurrent Markov chains II.

For notation see Part I*. Let f be a positive integer valued measurable function on S, and $U(w) = \sum_{N=1}^{\infty} Pr[\sum_{i=1}^{N} f(X_i) = w | X_0 = x]$. If $\int_S f(x)Q(dx) < \infty$ there exists a positive integer c and numbers $k_0,\ldots,k_{c-1}$ such that, as n tends to ∞, $U_x(nd + i)$ approaches $k_i, i = 0,\ldots,c - 1$, for each $x \in S$. Even in the acyclic case $c > 1$ is possible. It is shown that there exist $S_n \in \mathcal{B}$, $n = 1,2,\ldots$ such that $S_n \subseteq S_{n+1}$, $S = \bigcup_n S_n$ and whenever $A \subseteq S_n$ the process on A satisfies Doeblin's condition. These results are used to investigate asymptotic properties of suitably normed sums $\sum_{n=1}^{N} g(X_n)$, where g is a real valued function on S. (Received December 5, 1958.) * Part I will appear in the February 1959 Issue.

553-148. J. M. Osborn: A new class of neofields.

Let c be any fixed positive real number, θ any fixed angle, and define a new operation "$\circ$" on the complex numbers by letting $x \circ y = z$ whenever $\arg(y - x) - \arg(z - x) = \theta$ and $|z - x| = c|y - x|$ for three complex numbers x,y and z. If N is the set consisting of the complex numbers and the symbol "e" under "+" and "$\circ$", where the action of e is defined by the relations $e \circ x = x \circ e = x$, $e + x = x + e = e$ and $x \circ x = e$, then N is a neofield (with "+" as the upper operation). By picking c and θ appropriately and by applying this construction to tessellations and projective planes, neofields with a variety of different properties may be obtained. For example, if $c = 1$ and $\theta = \pi/3$, then the elements of N form a cross inverse property loop under "$\circ$". Using this remark, it can be shown that for any square-free odd integer $n > 1$ which is not divisible by a prime of the form $6m + 5$, there exists a simple cross inverse property loop of order $n + 1$. Examples of simple weak inverse property loops not satisfying the inverse property or the cross inverse property are also obtained. (Received December 5, 1958.)

553-149. Walter Rudin: Functions which operate on Fourier-Stieltjes transforms.

A complex function F, defined on a set E in the complex plane, is said to operate in a function algebra R if $F \circ f$ belongs to R for all f in R whose range lies in E. Let A(G) denote the algebra of all Fourier transforms on the infinite locally compact group G, and let B(G) be the algebra of all Fourier-Stieltjes transforms on G. Suppose F is defined on the interval $I = [-1,1]$, and $F(0) = 0$. Theorem 1 (G discrete): F operates in A(G) if and only if F is analytic in a

neighborhood of the origin. Theorem 2 (G not discrete): F operates in A(G) if and only if F is analytic on I. Theorem 3 (G compact): F operates in B(G) if and only if F is analytic on I. Theorem 4 (G not compact): F operates in B(G) if and only if F can be extended to an entire function in the complex plane. Special cases of these results were obtained recently by Katznelson, Helson, Kahane, and Rudin (C.R. Acad. Sci. U. S. A. vol. 247 (1958), July 28, August 11, September 8). The asymmetry of B(G), for noncompact G, is a corollary of Theorem 4. (Received December 5, 1958.)

553-150. S. M. Shah and S. K. Singh: Meromorphic functions with maximum defect.

Let f(z) be a meromorphic function of finite order ρ. If $\sum_1^\infty \delta(a_i) = 2$, $\delta(a_i) = 1$, ($|a_1| \leqq \infty$) then it is proved that ρ is integer. This result extends a theorem of Pfluger who showed that if f(z) be an entire function of finite order ρ with maximum defect 2 then ρ is integer. (Received December 5, 1958.)

553-151. Edward Silverman: A problem of least area.

One attack on the Plateau problem is to make use of an appropriate representation theorem (Morrey or McShane) to reduce the problem of minimizing the area integral to that of minimizing the Dirichlet integral. If x is continuous on a square Q into E_n let I(f,x) be the integral over Q of a continuous function f(x,J) which is positive for $J \neq 0$, positively homogeneous of degree one and convex in the jacobians. Then a new function $\bar{f}$ and nonparametric integral $I(\bar{f},x)$ are defined with the property that $I(f,x) \leqq I(\bar{f},x)$. A sufficiently smooth mapping for which the equality holds is called f-quasi-conformal. It is shown that a Fréchet surface has an f-quasi-conformal representation whenever it has a (Morrey) quasi-conformal representation. The method involves the introduction of f-Lebesgue and f-Peano areas, which are given by I(f,x), if x is smooth enough. The problem of minimizing $I(\bar{f},x)$ is straight-forward and, in conjunction with the representation theorem, is equivalent to minimizing I(f,x). The containing space can be any Banach space. (Received December 5, 1958.)

553-152. Andrew Sobczyk: Homogeneous fibre structures.

A relation $\sim$ in a set B is reflexive if its graph W in $B \times B$ contains the diagonal Δ, symmetric if $W^{-1} = W$, and transitive if $W \circ W = W$. A fibre structure is a set B, together with two reflexive and symmetric relations $\sim$

(tilda) and δ (delta), with respective graphs W, V, having the following properties: (i) each element of B belongs to at least one tilda class (maximal class of elements all mutually related by $\sim$) and to at least one delta class; (ii$\sim$) each tilda class contains exactly one representative of each delta class. Theorems: The structure B has property (iiδ). The structure B is a Cartesian product if and only if $\sim, \delta$ are both equivalence relations. Under additional hypotheses, the structure B has the characteristic property of the base classes and fibres of a fibre-bundle; at least one of $\sim$, δ is an equivalence relation. (Received December 5, 1958.)

553-153. Clifford Spector: Strongly invariant hierarchies. Preliminary report.

By a hierarchy of number-theoretic sets we mean a strictly increasing mapping from a set of ordinals or of ordinal notations to the set of subsets of the natural numbers partially ordered by "A is recursive in B". A hierarchy is called invariant if the sets corresponding to a given ordinal belong to the same degree, and strongly invariant if these sets are identical. The Davis-Kleene hyperarithmetical hierarchy defines degrees $\underset{\sim}{b}_a$ for each ordinal $a < \omega_1$. Similar hierarchies which are strongly invariant are obtained by determining the exact degrees of the sets O_a (= set of all notations in O for ordinals < a) and W_a (= set of all Gödel numbers of recursive well-orderings of the natural numbers of ordinal < a) for each $a < \omega_1$. If we set $\alpha(a) = \omega + \omega \cdot \omega^a$, $\beta(a) = \omega^a$, $\gamma(c) = 2c$, $\gamma(b\omega + c) = b\omega + 2c + 1$ ($a < \omega_1$, $0 < b < \omega_1$, c finite), then "$x \in O_{\alpha(a)}$" and "$x \in W_{\beta(a)}$" are complete predicates in existential quantifier form of degree $\underset{\sim}{b}_{\gamma(a)}$ for $0 < a < \omega_1$. The degree of O_a and of W_b for $\alpha(c) < a < \alpha(c + 1)$, $\beta(c) < b < \beta(c + 1)$, and $c < \omega_1$ is $\underset{\sim}{b}_{\gamma(c)+1}$. The degrees of the remaining sets O_a and W_b are easily determined. (Received December 5, 1958.)

553-154. C. T. Taam: On compact linear transformations in Banach space.

Let T be a compact linear transformation from a Banach space X to a Banach space B. The strongly closed unit sphere S^* in B^* is a compact Hausdorff space in the weak* topology. As a subspace of B^{**}, B can be embedded in the Banach algebra C of the complex-valued weak*-continuous functions in S^* such that the original strong norm of a vector in B is equal to its uniform

norm in S^*. Assume the embedding of B in C. It is shown that T^* is a weak*-continuous function from S^* to X^* and can be uniformly approximated arbitrarily close by finite sums of the form $h_k T^*(F_k)$, where F_k is in S^* and h_k in C. It follows that the compact linear transformation T can be approximated arbitrarily close in norm by degenerate linear transformations. (Received December 5, 1958.)

553-155. B. R. Vanderslice and D. W. Hall: Chromatic polynomials.

G. D. Birkhoff and D. C. Lewis, Jr. have made a conjecture (Trans. Amer. Math. Soc. vol. 60 (1946) pp. 355-451, in particular pp. 411-413) which, if true, will establish the four color conjecture in the affirmative. This paper builds up a great deal of experimental evidence favoring the conjecture. In particular, the chromatic polynomial for the truncated icosahedron is obtained explicitly. Chromatic polynomials not previously known for slightly fewer than fifteen hundred maps have been computed. These results mark the completion of the project announced in Abstract 57-184 (1951). The computation was greatly facilitated by the work of J. W. Siry (Abstract 490-438 (1953)). The completion of the work was made possible by a generous grant from the Research Foundation of the State University of New York. (Received December 5, 1958.)

553-156. R. F. Williams: A general inequality for a Lebesgue type area.

Suppose n is a positive integer, X is a compact Hausdorff space of dimension $\leqq n$, and that $f:X \to E_n$ is a light map. In this setting, a Lebesgue type, n-dimensional area $L_m^*(f)$, is defined, (Lebesgue area for compact two-dimensional surfaces, these Notices, vol. 5, no. 3, p.301). Define $M_f(p)$ to be the number, (possibly infinite) of points in $f^{-1}(p)$, for $p \in E_n$. We prove that $L_m^*(f) \leqq$ the lower Riemann integral of f over E_n. (In this setting M_f need not be measurable --it is if X has a countable base.) Ordinary Lebesgue area does not in general satisfy this inequality. A key technique: for every open cover U of X there is a refinement V such that f can be factored, $f = hg$, where $g:X \to X_V$ is canonical into the nerve of V, and $h:X_V \to E_n$ is simplicial relative to a subdivision of E_n. (Received December 5, 1958.)

553-157. J. W. Woll, Jr.: Subordination of infinitely divisible processes.

For the definition of subordination see Bochner, Harmonic analysis and the theory of probability, University of California Press, Chapter 4, section 4.3.

A (b,A,F) - process is an infinitely divisible process on R^n whose transition probabilities P(t,0,B) have Fourier transform $\exp\{t[ixb - xAx + \int f(x,y)F(dy)]\}$, where xb is the inner product of x and b, $f(x,y) = e^{ixy} - 1 - ixy(1 + \|y\|^2)^{-1}$ and b, A and F have the usual interpretation. Theorem: If a process is subordinate to a (b,A,F) - process, it is a $(\zeta + cb, cA, cF + H)$ - process, where $H(B) = \int N(ds)P(s,0,B)$, N is a Borel measure on $(0, +\infty)$ bounded in the sense that $\int s(1 + s)^{-1}N(ds) < +\infty$, $\zeta = \lim_{\epsilon \to 0} \int_\epsilon^{+\infty} N(ds) \int x(1 + \|x\|^2)^{-1}P(s,0,dx)$, and $c \geq 0$. Two processes are said to be of the same type if they differ by a linear change of the time parameter. Theorem: A (b,A,F) - process is subordinate only to processes of the same type if (i) support (F) is compact, or (ii) if $A \neq 0$ and the projection of support (F) on the range of A is not onto. This shows that many types are maximal under subordination in the sense of Theorem 4.3.2 of the above reference. (Received December 5, 1958.)

553-158. Oved Shisha: Nearest functions.

Let S be a set of complex numbers, Π a complex vector space of complex functions defined in S and f(z) a complex function defined in S. A point p(z) of Π will be called a nearest function to f(z) on S with respect to Π, if there does not exist a q(z) ($\in \Pi$) satisfying: (a) $q(z) \not\equiv f(z)$ on S; (b) if $z \in S$ and $p(z) \neq f(z)$, then $|f(z) - q(z)| < |f(z) - p(z)|$, (c) if $z \in S$ and $p(z) = f(z)$, then $q(z) = f(z)$. Theorem: Let S be a nonempty compact set of complex numbers, $f(z), p_1(z), p_2(z), \ldots, p_n(z)$ complex functions, defined and continuous in S, Π the set of all functions expressible throughout S in the form $\sum_{\nu=1}^{n} c_\nu p_\nu(z)$ (c_ν complex), and p(z) ($\in \Pi$) a nearest function to f(z) on S with respect to Π such that $p(z) \neq f(z)$ throughout S. Then there exist distinct points $z_1, z_2, \ldots, z_m$ ($\in S$), (*) $m \leq 2n + 1$, and positive $\lambda_1, \lambda_2, \ldots, \lambda_m$ such that p(z) is a nearest function to f(z) on $\{z_1, z_2, \ldots, z_m\}$ with respect to Π, and such that $\sum_{\mu=1}^{m} \lambda_\mu p_\nu(z_\mu) \cdot [f(z_\mu) - p(z_\mu)]^{-1} = 0$ ($\nu = 1,2,\ldots,n$). In certain cases (*) can be improved. The above generalizes a definition and a theorem of Fekete (Bull. Res. Council of Israel, Sect. A. vol. 5 (1955) pp. 11-19) whose method of proof is used. (Received December 5, 1958.)

553-159. Louis Sucheston: On sequences of events.

A sequence of events A_i is called equidistributed of order r and with density p if for each event M each subsequence of A_i contains a further subsequence B_i such that $\lim_{n_i \to \infty} P_M(B_{n_1} \ldots B_{n_r}) = p^r$, $n_i \neq n_j$ if $i \neq j$ (P_M is the

conditional probability given M, $\lim_{n_i \to \infty}$ the limit when all indices n_i converge independently to infinity). A sequence A_i is called condensed of order r and with density p if for some $\delta > 0$ $P(A_{n_1} \ldots A_{n_r}) \geq p^r + \delta$, $n_1, \ldots, n_r$ $1,2,\ldots$; semi-condensed if it contains a condensed subsequence. Theorem C. If A_i is a sequence of events such that $\lim_{n_i \to \infty} P(A_{n_1} \ldots A_{n_r}) \geq p^r$ then exactly one of the following two conditions (α), (β) is satisfied: (α) The sequence A_i is equidistributed with density p of orders $s = r, r+1, \ldots$. (β) The sequence A_i is semi-condensed with density p of orders $s = r+1, r+2, \ldots$. This theorem contains as a special case a theorem of Renyi (Theorem 2, Acta Mathematica, Budapest, vol. 9 (1958) pp. 215-228); it was obtained independently and by different methods. Paper to appear in Journal London Math. Soc. (Received December 5, 1958.)

553-160. Gabriel Margulies: The differential invariants of local regular surface mappings.

A local homeomorphism between two surfaces embedded in E_3 can be determined by a tensor and a vector, called mapping tensors; these are objects in Riemann space R_2 and permit the development of an intrinsic mapping theory for surfaces embedded in E_3. The systematic covariant use of mapping tensors, together with the use of nonholonomic frames determined by their eigen-directions opens a new way for the investigation of properties of pairs of surfaces and their joint differential invariants. Starting from the integrability conditions which the mapping tensors must satisfy, a fundamental existence theorem is proved for pairs of surfaces and their relative mapping in E_3; the existence criteria are in the form of partial differential equations analogous to the classical equations of Gauss-Mainardi-Codazzi. Also, if certain isometry conditions be added to the initial homeomorphism, the classical infinitesimal deformation theory follows as a special case of this mapping theory. The mapping theory leads also to the solution, in intrinsic form, of the characterization problem for surfaces with systems of joint differential invariants. The solution is obtained by solving the differential equations representing the integrability conditions for the mapping tensors in special nonholonomic frames determined by the eigen-directions of the mapping tensors. (Received December 3, 1958.)

ERRATA, Volume 5

B. A. Fleishman, Progressing waves in an infinite nonlinear string. Preliminary report.

Pages 239-240, Abstract 546-11. The next to the last sentence should read, "If $\beta < 0$, at least two types of waves exist: (a) small-amplitude periodic waves similar to those occurring when $\beta > 0$; (b) a class of nonperiodic solutions."

INDEX OF ABSTRACTS

VOLUME 5

(Number refers to page on which abstract begins.)

Abel, W. R. and Blumenthal, L. M. Distance geometry of metric arcs. II, 90.

Abhyankar, Shreeram. Fundamental groups of algebraic manifolds, 189; Local fundamental groups of Kroneckerian varieties, 235.

Abian, Smbat and Brown, A. B. A note on the solution of the differential equation of the type $g(x,y,y') = 0$, 245; On the solution of an implicit equation, 572; On the solution of simultaneous first order implicit differential equations, 660; On the solution of an nth order implicit differential equation, 774; On the solution of simultaneous implicit equations, 786.

Adams, J. F. On the nonexistence of elements of Hopf invariant one mod p, 25.

Addison, J. W. Some consequences of the axiom of constructibility, 845.

Adler, A. W. and Kohn, J. J. On differentiability properties of Kahler metrics, 790.

Agmon, Shmuel. On coercive Dirichlet forms, 59.

Aissen, M. I. Existence and convergence of formal solutions of certain systems of differential equations. Preliminary report, 502.

Alavi, Yousef and Wells, C. P. Expansions of parabolic wave functions, 811.

Alling, N. L. On an extension of a classical theorem on groups, 693; On certain totally ordered non-abelian groups of rank 2, 803.

Amemiya, Ichiro and Halperin, Israel. Finiteness and continuity in complemented modular lattices, 323.

Amir-Moez, A. R. Some equalities in a unitary space leading to equalities concerning singular values of sets of matrices, 236.

Amitsur, S. A. Means of arithmetic functions, 831.

Anderson, A. G. The prediction of quantitative characteristics in polygenic systems. III, 304.

Anderson, F. W. Concerning a generalization of the Stone-Weierstrass approximation theorem. Preliminary report, 70; A class of function algebras, 379.

Anderson, F. W. and Blair, R. L. Characterizations of the algebra of all real-valued continuous functions on a completely regular space, 71; Spaces of real ideals, 378.

Anderson, L. W. and Bharucha-Reid, A. T. Topological semigroups and n-means, 67.

Anderson, R. D. A certain two-dimensional continuum, 79; The groups of all homeomorphisms of certain homogeneous spaces, 305; Minimal sets under flows on tori and derived spaces, 843.

Andrushkiw, J. W. A note on monotonely decreasing functions, 334.

Anselone, P. M. Integral equations of the Schwarzchild-Milne type, 45; Integral equations of radiative transfer, 305; Ergodic theorems for discrete semi-Markov processes. Preliminary report, 832.

Appel, K. I. Horn sentences in identity theory, 343.

Aronson, D. G. On the fundamental solution of a parabolic system of equations. Preliminary report, 495.

Artzy, Rafael. On automorphic-inverse properties in loops, 572.

Asenjo, F. G. On the formalization of the concept of internal relation, 832.

Askovitz, S. I. Some applications of centroids to problems in numerical analysis, 832.

Aurora, Silvio. Multiplicative norms for metric rings. I, 337; Multiplicative norms for metric rings. II, 338; Multiplicative norms for metric rings. III, 338; On power multiplicative norms. I, 338; On power multiplicative norms. II, 339; On certain regular metric rings, 339.

Auslander, Louis. Locally affine spaces with abelian holonomy groups, 151; A class of discrete uniform subgroups of solvable Lie groups, 348.

Austin, D. G. On the interdependence of the Kolmogorov differential equation, 220; Some results in Markov chain theory, 490; On the entrance-boundary for Markov processes with discrete state

space, 793.
Axt, Paul. On a subrecursive hierarchy and primitive recursive degrees, 164.
Ayoub, R. G. Ideals and the Dedekind zeta function, 589.
Aziz, A. K. On a higher order boundary value problem for hyperbolic partial differential equations, 572.
Bade, W. G. Similarity equivalence of spectral operators, 369; Multiplicity of spectral measures. II, 370; A class of non-self-adjoint boundary conditions for the Laplacian. Preliminary report, 843.
Bade, W. G. and Curtis, P. C., Jr. Continuity properties of homomorphisms of function algebras. Preliminary report, 833.
Baer, R. M. and Brock, Paul. Natural sorting, 225.
Bagley, R. W. and McKnight, J. D., Jr. On Wallman's method, 199.
Bailey, J. L. Thermal shock on the surface of the elastic half-space, 208.
Baker, G. A., Jr. and Oliphant, T. A. An implicit, numerical method for solving the two-dimensional, heat equation, 462.
Ball, B. J. The sum of two solid wild 2-spheres, 305; Certain collections of arcs in E^3, 320.
Bareiss, E. H. Modified Hessenberg method, 306.
Barrett, J. H. Nonoscillation of second-order linear differential equations with non-negative coefficients, 64;
——See Coles, W. J.
Barrett, L. K. On n-regular curves, 228.
Bartle, R. G. On the openness and inversion of differentiable mappings, 786.
Bass, R. W. The periodic solutions of the n-body problem, 845.
Bauer, F. L. and Frank, Evelyn. Note on formal properties of certain continued fractions, 57.
Baum, W. R. Integral geometric methods in information theory. I. Invariant measures, 819.
Baxter, G. E. On the number of changes in sign, 833.
Bear, H. S. A strong maximum modulus property for maximal function algebras, 359; Some boundary properties of maximal algebras, 676.
de Beaumont, Henry and Martinek, Johann. Research into the sets of solutions of certain systems of nonlinear partial differential equations of higher order, 211.
Beaumont, R. A. Generalized rings, 323.
Beaumont, R. A. and Wisner, R. J. The nucleus of a torsion-free abelian group, 676; Integral domains defined on torsion-free groups of rank two, 689.
Beckenbach, E. F. Subfunctions and inequalities, 200.
Bell, C. B. On the structure of stochastic independence, 353.
Beneš, V. E. On exponential holding-time II. Continuous time treatment for a trunk group serving a renewal process, 187; On exponential holding-time IV. The character of overflow traffic, 187; Characterization and decomposition of stochastic processes with stationary independent increments, 246; The general queue with one server, 491; Type I counters with arbitrary particle arrivals, 772; Combinatory methods in the general theory of queues, 774; General stochastic processes in models of telephone traffic and Type I counters, 819.
Berberian, S. K. Note on a theorem of Fuglede and Putnam, 479.
Bergman, Stefan. Operators generating solutions of certain partial differential equations. I, 331; Operators generating solutions of certain partial differential equations. II, 332; A class of pseudo-conformal and quasi-pseudo-conformal mappings, 363.
Berkovitz, L. D. and Dresher, Melvin. Solution of a multimove tactical game, 502.
Bernstein, Barry. A maximum theorem for irrotational subsonic gas flow, 794.
Bernstein, Barry and Ericksen, J. L. Work functions in hypo-elasticity, 166.
Beyer, W. A. The divergence set of Rademacher series, 695; Rademacher series with geometric coefficients. Preliminary report, 842.
Bharucha-Reid, A. T. See Anderson, L. W.
Bing, R. H. A surface S is tame in E^3 if E^3 - S is locally simply connected at each point of S, 180; The cartesian product of a certain nonmanifold and a line is E^4, 244; A 2-manifold-with-boundary in E^3 is tame if its complement is 1-ULC, 365; Each homogeneous plane continuum that contains an arc separates the plane, 696.
Bing, R. H. and Curtis, M. L. Imbedding decompositions of 3-space in 4-space,

685.
Birkhoff, Garrett and Varga, R. S. Implicit alternating direction methods, 63.
Blackman, Jerome and Pollard, Harry. The finite convolution transform, 236.
Blair, R. L. See Anderson, F. W.
Blattner, J. W. and Paige, L. J. An identity for Jordan algebras. Preliminary report, 573.
Blattner, R. J. Automorphic group representations, 226.
Bleicher, M. N. See Preston, G. B.
Block, Richard. Relations between torsion-free abelian groups and certain Lie algebras, 323; On Lie algebras of rank one, 605.
Blum, E. K. and Curtis, P. C., Jr. An asymptotic relation between Tschebycheff coefficients and the best polynomial approximation, 503.
Blumenthal, L. M. See Abel, W. R.
Bolie, V. W. Divergence from Bessel's integral in a radiation equation, 223.
Boothby, W. M. and Wang, H. C. Homogeneous contact manifolds, 193.
Bourgin, D. G. Maps of spheres, 576.
Bourne, S. G. On ideal theory in a commutative semigroup, 200.
Bourne, S. G. and Zassenhaus, H. J. On the semiradical of a semiring, 333.
Boyer, D. L. and Walker, E. A. Almost locally pure Abelian groups. Preliminary report, 196.
Brace, J. W. Almost uniform convergence versus pointwise convergence, 819.
Bradford, J. C. The Banach-Blumberg property and Blumberg's theorem, 168.
Bragg, L. R. See Bureau, F. J.
Brainerd, B. and Lambek, Joachim. Rings of quotients of Boolean rings, 601.
Bram, Joseph. Chebychev approximation in locally compact spaces, 21.
Bramble, J. H. Continuation of biharmonic functions across circular arcs, 167.
Bratton, Donald. New results in the theory and techniques of Chebyshev fitting, 248; A theorem on factorization in semigroups that results from a theorem of Kleene's in the theory of automata, 367; A machine process for maximizing a function of one real variable, 367; Concrete representation of the axiom of choice within Boolean algebra, 367; A fundamental theorem in the theory of continued fractions, 368; The theory of automata, 470.
Brauer, A. T. and Mewborn, A. C. Intervals for the characteristic roots of an Hermitian matrix, 80.
Brauer, F. G. Some results on uniqueness and successive approximations, 573.
Brauer, F. G. and Sternberg, Shlomo. Local uniqueness, existence in the large, and the convergence of successive approximations, 306.
Braunschweiger, C. C. Interior-like elements of cones. Preliminary report, 684.
Brind, Abraham. Conclusive elementary proof of Fermat's last theorem for all odd $n \geqq 3$, 495.
Brinkman, K. L. and Deverall, L. I. Application of group representation theory to determination of natural frequencies of vibrating plates, 464.
Brock, Paul. See Baer, R. M.
Browder, William. Loop spaces of suspensions. I, 370; Loop spaces of suspensions. II, 461; Suspensions in loop spaces, 820.
Brown, A. B. See Abian, Smbat.
Brown, Leon, Shields, A. L. and Zeller, Karl. On absolutely convergent exponential sums, 820.
Brown, R. K. Typically-real functions, 574.
Brownell, F. H. Explicit perturbation formulae and convergence theorems, 804.
Bruck, R. H. Some theorems on Moufang loops, 816.
Brunk, H. D. Uniform consistency of estimators of linearly ordered parameters, 92.
Büchi, J. R. and Elgot, C. C. Decision problems of weak second order arithmetics and finite automata. Preliminary report. Part I, 834.
Büchi, J. R., Elgot, C. C. and Wright, J. B. The nonexistence of certain algorithms of finite automata theory. Preliminary report, 98.
Buck, C. C. A pair of mutually independent dual theorems in projective geometry, 503.
Buck, R. C. The Riesz property of a partially ordered linear space, 31; The algebra of bounded analytic functions, 31; A Weierstrass theorem for function modules, 169.
Bureau, F. J. and Bragg, L. R. The solutions of a system of partial differential

equations in a region of hyperbolic normality. Preliminary report, 324.
Burgess, C. E. Homogeneous plane continua which are circularly chainable and contain an arc, 229; Some conditions under which a homogeneous continuum is a simple closed curve, 675.
Burke, C. J. and Rosenblatt, Murray. A Markovian function of a Markov chain, 182.
Burrow, M. D. Invariants of a free Lie ring, 32.
Bushaw, D. W. The equation of damped vibrations, 358.
——See Firey, W. J.
Butler, J. B. Jr. A note on perturbation theory of ordinary differential operators. Preliminary report, 692.
Butson, A. T. Sufficient conditions for an abstract group to be ordered. Preliminary report, 329.
Cahn, Albert and Straus, E. G. On piecewise congruence. Preliminary report, 360.
Calabi, Eugenio. Improper affine hyperspheres of convex type and a generalization of a theorem by K. Jörgens, 93.
Caldwell, W. V. Maximal vector spaces of light, interior, orientation-preserving C^1 functions, 821.
Cameron, R. H. and Donsker, M. D. Inversion formulae for characteristic functionals of stochastic processes, 186.
Cargo, G. T. Boundary behavior of Blaschke products, 805.
Carlitz, Leonard. Some finite summation formulas of arithmetic character, 781.
Carroll, Frank. On a class of noncontinuable analytic functions, 351.
Carroll, R. W. On the EPD equation and subharmonic distributions. Preliminary report, 455.
Case, J. H. Some snake-like continua. Preliminary report, 199.
Cell, J. W. Closed-form solutions of $z''+(p+u)z'+(q+v)z=0$ in terms of solutions of $y''+py'+qy=0$ and of known functions, 691.
Cesari, Lamberto. Quasi additive set functions and the concept of integral, 609; On total stability, 690.
Cesari, Lamberto and Neugebauer, C. J. On the coincidence of Geöcze and Lebesgue areas, 195.
Chak, A. M. A class of polynomials analogous to Lommel polynomials, 58.
Chamberlin, R. E. A class of quasi-complexes, 302.
Chang, C. C. Groups which are automorphism groups of a simply ordered set. Preliminary report, 491.
Charnes, Abraham and Cooper, W. W. Extremal principles for simulating traffic flow in a network, 592; The strong Minkowski-Farkas-Weyl theorem for vector spaces over ordered fields, 594.
Charnes, Abraham and Lemke, C. E. The continuous-limit method I. Minimization of convex functionals over convex polyhedra, 593; Multi-copy generalized networks, 660.
Chen, T. C. A modified power method for finding the eigenvalues and vectors of matrices. Preliminary report, 370.
——See Willoughby, R. A.
Chen, Y. W. Hoelder continuity and initial value problems of mixed type differential equations, 821.
Cheney, Ward. Power series for the n-body problem, 795.
Chessin, P. L. On the reversion of power series, 481.
Chover, Joshua. Conditions on the realization of prediction by measures, 175; A theorem on integral transforms with an application to prediction theory, 798.
Chow, Y. S. Pointwise convergence of martingales with a directed index set, 806.
Chowla, Paromita. Diophantine equations in quadratic number-fields, 818.
Chowla, Sarvadaman. See Jones, B. W.
Christensen, D. J. See Pierce, R. S.
Chu, Hsin. Bonded groups, 844.
Church, P. T. On cluster sets in n-dimensional space, 206; On global cluster sets of meromorphic functions, 472; A Riemann mapping theorem for quasi-conformal maps on certain domains in E^3, 574; Boundary images of meromorphic functions, 603.
Clark, C. W. Asymptotic distribution of the eigenvalues for the lower part of the one-dimensional Schrödinger operator spectrum, 227; Asymptotic distribution of the eigenvalues for the lower part of the n-dimensional Schrödinger operator spectrum, 362.
Coburn, Nathaniel. The method of characteristics for a perfect compressible fluid in general relativity and nonsteady Newtonian mechanics, 62.

Coburn, Nathaniel and Ong, R. S. B. General and transverse waves in hydromagnetic flows, 774.
Cockcroft, W. H. The cohomology groups of certain fibre spaces, 574.
Cohen, Eckford. Representations of even functions (mod r) II. Cauchy products, 152; A class of residue systems (mod r) and related arithmetical functions, 157; Representations of even functions (mod r). III. Special topics, 336; Some remarks on Galois theory, 459; Arithmetical inversion formulas; 459; The characteristic divisors of a polynomial function of a matrix, 575; A class of residue systems (mod r) and related arithmetical functions, II. Higher dimensional analogues, 575; The Brauer-Rademacher identity, 576; A trigonometric sum, 660; A class of arithmetical functions of several variables, I. Applications to congruences, 781; Arithmetical functions associated with arbitrary sets of integers, 781; Asymptotic averages in a class of arithmetical functions of two variables, 782; Inversion of primitive functions (mod r) and Nagell's totient function. 786.
Cohen, H. J. See Supnick, Fred.
Cohen, Haskell and Krule, I. S. Continuous homomorphic images of real clans with zero, 359.
Cohen, P. J. On Green's theorem, 250; Factorization in group algebras, 347; On the L^1 norm of exponential sums, 791.
Coleman, C. S. Integral curves of a certain class of differential systems, 374; Stable conal regions in the neighborhood of a singular point in three space, 465.
Coles, W. J., Barrett, J. H. and Wend, D. V. V. Nonoscillation and disconjugacy in the complex plane. Preliminary report, 503.
Comfort, W. W. Šilov boundaries induced by certain Banach algebras, 226.
Connell, E. H. Products of fixed point spaces, 504.
Conner, P. E. and Dyer, Eldon. Singular fiberings by spheres, 303.
Conrad, P. F. The groups of word preserving permutations of a group, 84; Halfgroupoid rings, 355.
Cooke, K. L. The rate of increase of real continuous solutions of certain algebraic functional equations, 74.
Cooper, W. W. See Charnes, Abraham.
Copeland, A. H., Jr. See Copeland, A. H., Sr.
Copeland, A. H., Sr. and Copeland, A. H., Jr. Convex geometry, 330.
Correl, Ellen. On a class of topologies for division rings. Preliminary report, 97.
Cowling, V. F. and Royster, W. C. On the Euler and Taylor summation of Dirichlet and Taylor series, 844.
Coxeter, H. S. M. Twelve points in PG(5, 3) with 95040 self-transformations, 362; Close-packing and froth, 822.
Crouch, R. B. Characteristic subgroups of monomial groups, 196.
Cunkle, C. H. and Utz, W. R. Equicontinuous and related flows, 817.
Curtis, C. W. On the irreducible representations of certain Lie algebras of prime characteristic, 306; Quasi-Frobenius rings and Galois theory, 320.
Curtis, M. L. See Bing, R. H.
Curtis, M. L. and Fort, M. K., Jr. The fundamental group of one-dimensional spaces, 84; Singular homology of one-dimensional spaces, 782.
Curtis, P. C., Jr. n-parameter families of functions and best approximation, 496.
——See Bade, W. G.; Blum, E. K.

Dade, E. C. and Goldberg, Karl. Incidence algebras, 41.
Daigneault, Aubert. The free product of polyadic algebras and applications, 799.
Danese, A. E. Some identities and inequalities involving ultraspherical polynomials, 246.
Dantzig, G. B. A decomposition principle for linear programs, 811.
Davis, Chandler. The set of nonlinearity of a convex piecewise-linear function, 241; Various averaging operations onto subalgebras, 476; On functional calculus, 844.
Davis, Martin. See Putnam, Hilary.
Davison, W. F. Cartesian products of mosaic spaces, 168.
Dawson, D. F. Continued fractions with absolutely convergent even or odd part, 89; Concerning a property of continued fractions, 343; Convergence of Stieltjes type continued fractions, 470.
DeCicco, John. The geometry of the z-plane based on a quadratic extension Γ of a field K, 151; The group of bilinear transformations of a quadratic extension

Γ of a field K, 342.
DeHardt, J. H. A theorem on the convergence of empiric measures, 91.
Dekker, J. C. E. The factorial function for isols, 49.
Dengler, M. A. The decay of a vortex under the influence of viscosity, 21; Derivative coefficients for the numerical solution of partial differential equations, 77.
Dennemeyer, R. F. Elliptic forms in Hilbert space, and second order elliptic differential equations, 224.
Deskins, W. E. On endomorphisms of nonabelian groups, 347; An extension of a theorem of Herstein, 576; Some generalized Frattini subgroups of finite groups. Preliminary report, 834.
Deverall, L. I. See Brinkman, K. L.
Devinatz, Allen and Hirschman, I. I., Jr. The spectra of multiplier transforms on l^p, 40; Multiplier transforms on $l^{2,\alpha}$, 40.
De Vogelaere, Reni. Over-relaxations, 147.
Dickson, R. J. A model of turbulence displaying autonomous oscillations. Preliminary report, 822.
Dixon, J. D. Properties of the coefficients of cyclotomic polynomials, 160.
Dolph, C. L. and Penzlin, F. On the theory of a class of non-self-adjoint operators and its applications to quantum scattering theory, 788.
Donsker, M. D. See Cameron, R. H.
Dorff, E. K. On Banach algebras of infinite matrices, 485.
Douglas, Jim, Jr. and Gallie, T. M., Jr. An approximate solution of an improper boundary value problem, 491.
Douglis, Avron. On a boundary problem for a quasi-linear parabolic equation, 318; On discontinuous initial value problems for certain nonlinear partial differential equations, 369; An extended ordering principle for generalized solutions of a quasi-linear partial differential equation, 504.
Dresher, Melvin. See Berkovitz, L. D.
Dunn, R. L. Set-valued functions with and without fixed points. Preliminary report, 475.
Dwinger, Philip. Completeness of quotient algebras of Boolean algebras. II, 159; Completeness of quotient algebras of Boolean algebras. III, 692; Retracts in Boolean algebras, 797.
Dwork, B. M. Norm residue symbol in cyclotomic fields, 22.
Dyer, Eldon. On the dimension of products, 302; Duality theorems for generalized manifolds, 302; The local Künneth sequence, 303.
——See Conner, P. E.

Eberlein, W. F. Integrals over function space. III, 250; Integrals over function space. II. Preliminary report, 307.
Edmondson, D. E. A modular topological lattice, 92.
Edrei, Albert and Fuchs, W. H. J. A gap theorem for entire functions with deficient values, 200; A sharpened form of Picard's theorem for meromorphic functions of finite order, 371; On meromorphic functions of order less than one, 496; A property of entire functions with negative zeros, 497; On the growth of meromorphic functions, 606.
Eells, James, Jr. A generalization of the Gauss-Bonnet theorem, 177.
Ehrenfeucht, Andrzej. Theories having at least continuum many nonisomorphic models in each infinite power, 680.
Eisenberg, Edmund and Gale, David. Aggregation of utility functions, 465.
Elgot, C. C. and Wright, J. B. Series-parallel graphs and lattices 71.
——See Büchi, J. R.
Ellis, D. O. Remarks on the elementary symmetric functions, 90; Some consequences of Wilson's theorem, 683.
Enochs, Edgar. p-closed groups, 324.
Epstein, Bernard. The kernel-function and conformal invariants, 601.
Erdös, Paul and Lorentz, G. G. On the probability of relatively prime values of a function, 214.
Ericksen, J. L. See Bernstein, Barry.
Esser, M. H. M. A characterization of L^p spaces, 21.
Etter, M. J. On idempotent stochastic matrices. Preliminary report, 576.
Evans, Trevor. Substitution algebras. I, 307.
Evans, Trevor and Hardy, F. L. Sheffer stroke functions in many-valued logics, 81.
Evans, Trevor and Schwartz, P. B. On Slupecki T-functions, 42.

Fadell, Edward. An extension of a theorem of A. Dold and R. Thom, 343; On fiber homotopy equivalence, 787.
Faith, C. C. On conjugates in division

rings, 329; Galois extensions in which every element with regular trace is a normal basis element. I, 330; Galois extensions in which every element with regular trace is a normal basis element. II, 331; Concerning noninvariant submodules of simple, matrix, and local matrix rings, 373; On Abelian Galois groups, 497; Concerning noninvariant submodules of simple, matrix, and local matrix rings. II, 846.

Federer, Herbert. A note on the Gauss-Green theorem, 43; An integral formula, 167; A general integralgeometric formula, with application to curvatures. Preliminary report, 235; On sets with positive reach, applied to curvature theory. Preliminary report, 457.

Feeman, G. F. and Hsiung, C. C. Characterizations of Riemann n-spheres, 477.

Feferman, Solomon. Arithmetically definable models of formalized arithmetic, 679.

Feit, Walter. p-Extensions of p-adic number fields, 85.

Feldman, Jacob. Two bizarre connected spaces, 308.

Feldzamen, A. N. Similarity and unitary equivalence for normal operators, 95.

Fife, Paul and Isaacson, Eugene. Boundary-layer phenomena in the nonlinear bending of thin circular plates and shells. I, 772.

Findlay, G. D. and Lambek, Joachim. Rational extensions of modules, 51; A generalized ring of quotients. II, 247; Multiplicative ideal theory of a noncommutative ring, 606.

Firey, W. J. and Bushaw, D. Invariant arcs of certain mappings having two fixed points, 358.

Fitch, F. R. Representation of sequential circuits in combinatory logic, 247.

Flanagan, Czerna. See Maxfield, J. E.

Flanagan, Czerna and Maxfield, J. E. Singular perturbations of polynomials in several variables. Preliminary report, 504.

Flatt, H. P. Infinitely differentiable solutions of ordinary differential equations, 681.

Fleischer, Isidore. Best group alphabets, 796.

Fleischer, Isidore and Kooharian, Anthony. On the statistical treatment of stochastic processes, 182.

Fleishman, B. A. Progressing waves in an infinite nonlinear string. Preliminary report, 239, 855.

Fleming, W. H. Nondegenerate surfaces and fine-cyclic surfaces, 198; Sets with finite perimeter. Preliminary report, 308.

Fort, M. K., Jr. One-to-one mappings onto the Cantor set, 783; Images of plane continua, 783.

——See Curtis, M. L.

Fort, M. K., Jr. and Hedlund, G. A. Minimal coverings of pairs by triples, 577.

Fort, M. K., Jr. and Horstman, W. G. Topological completeness of a space of homeomorphisms, 688.

Fort, Tomlinson. A problem of Richard Bellman, 82; A problem in linear differential equations, 684.

Foster, B. L. Differentiation on manifolds without a connection, 349; Covariant differentiation, 378.

Foulis, D. J. Induced isomorphisms of tensor products. Preliminary report, 79.

Fox, Charles. Fractional integration and chain transforms, 246.

Fox, D. W. Uniqueness theorems for singular Cauchy problems. Preliminary report, 341.

Fox, W. C. Differentiation of Riemann surfaces, 835.

Frank, Evelyn. A note on certain determinantal equations, 163; A new class of continued fraction expansions for the ratios of hypergeometric functions. II, 380; New continued fraction expansions for the ratios of Heine functions, 473;

——See Bauer, F. L.

Frankel, T. T. Fixed points and torsion on Kähler manifolds, 368.

Frayne, T. E. and Scott, D. S. Model-theoretical properties of reduced products, 675.

Frayne, T. E., Morel, A. C. and Scott, D. S. Set-theoretical properties of reduced products, 674.

Frayne, T. E., Scott, D. S. and Tarski, Alfred. Reduced products, 673.

Freeman, R. S. Self adjoint boundary conditions for the Laplacian. I. Bounded regions in the plane, 661; Self adjoint boundary conditions for the Laplacian. II. Unbounded regions in the plane, 661.

Friedland, Bernard and Stern, T. E. On periodicity of states in linear modular

sequential circuits, 696.
——See Stern, T. E.
Friedman, Avner. Linear partial differential systems with an additional differential equation, 155; Harnack inequality for parabolic equations and its applications to the Dirichlet problem, 155; Existence and stability theorems for parabolic equations, 165; Free boundary problems for parabolic equations, 793.
Frisch, H. L. See Miranker, W. L.
Fuchs, W. H. J. See Harrei, Albert.
Fulks, W. B. and Maybee, J. S. On a nonlinear parabolic equation, 823.
Fullerton, R. E. Support properties for cones with quasi-interior points, 309.
Fulton, C. M. Linear completeness and hyperbolic trigonometry, 361.
Gaddum, J. W. Projective convexity in Euclidean space, 309.
Gál, I. S. On the completion of uniform structures, 72; Remarks on (m, n)-compact spaces, 249; On the topological foundation of the theory of distributions, 467; Distributions on measure spaces, 577; Multiplication and division of distributions, 598; Some properties of the Dirac delta, 662.
Gale, David. Transient flows in networks, 226.
——See Eisenberg, Edmund.
Gallie, T. M., Jr. See Douglas, Jim, Jr.
Garsia, A. M. Calculation of conformal parameters of imbedded Riemann surfaces, 241.
Gelbaum, B. R. Tensor products of Banach algebras, 472.
Gerstenhaber, Murray. On deformations of compact complex manifolds, 325; On nilalgebras and linear varieties of nilpotent matrices. III, 480; Nilalgebras and linear varieties of nilpotent matrices. IV, 806.
Getoor, R. K. An analogue of Mercer's theorem, 56; Markov operators and their associated semi-groups, 238; A class of integral transforms suggested by probability theory, 795.
Ghaffari, Abolghassem. On the asymptotic behavior of the integral curves of a certain nonlinear differential equation, 237; The behavior of Rayleigh's equation at infinity, 812.
Gilbert, R. C. Generalized resolvents of a symmetric operator, 228.
Gillman, Leonard and Jerison, Meyer. Homomorphisms of rings of continuous functions, 229; Closed sets in Stone-Čech compactifications, 488; Prime ideals in rings of continuous functions, 488.
Glasser, M. L. A note on integral equations and transforms. Preliminary report, 148.
Glicksberg, I. L. Convolution semigroups of measures, 379.
Goffman, Casper. Hausdorff measure and Lebesgue area, 591.
Goffman, Casper and Waterman, Daniel. On upper and lower limits in measure, 812.
Goldberg, Karl. See Dade, E. C.
Goldberg, S. I. A theorem on the characterization of spheres, 309.
Goldberg, Seymour. Linear operators and their conjugates, 225.
Goldman, A. J. and Stone, J. J. Continuous poker, 27.
Goldstein, A. A. Taylor series for the regularized N-body problem, 803.
Goldstine, H. H. See Horwitz, L. P.
Golomb, Michael. Existence and expansion theorems for periodic and almost periodic solutions of nonlinear differential systems, 325.
Golomb, S. W. A connected topology for the integers, 73.
Gomory, R. E. Solving linear programs in integers, 505.
Gorman, C. D. Brownian motion of rotation, 796.
Gosselin, R. P. On the divergence of Fourier series, 93; A theorem on L^p Fourier series, 242; On Diophantine approximation and trigonometric interpolating polynomials, 607.
Grace, E. E. Totally nonconnected im kleinen continua, 181.
Gray, J. W. Contact manifolds, 248.
Green, L. W. On a problem of Blaschke, 805.
Green, Simon and Rutledge, W. A. Regular transformations in Euclidean 4-space, 800.
Greenberg, Leon. Geometric proofs of two theorems in group theory. Preliminary report, 597; A discreteness theorem for subgroups of the Lorentz group, 662; Hyperbolic space forms. Preliminary report, 663.
Greendlinger, Martin. Dehn's algorithm for the word problem, 371.

Greenleaf, Newcomb and Wisner, R. J. On unique factorization in semigroups of the integers, 219.

Greenspan, Donald. On a "best" five point difference analogue of Laplace's equation, 156; On nine point analogues of Laplace's equation, 156; On a nine point method for the numerical evaluation of the Stokes' stream function, 156; On the numerical solution of Poisson's equation with given boundary values, 457; On the numerical solution of Dirichlet problems, 578.

Greenspan, Donald, Matsuno, Shozo and Ulrick, Bruno. On an analog solution of two point boundary value problems, 155.

Greenstein, D. S. Some new external problems with infinitely many interpolating conditions, 790.

Greever, J. J. Stationary points for finite transformation groups, 469.

Griffin, J. S., Jr. and McLaughlin, J. E. On the projective groups of lines, 23.

Griffith, H. C. Tame surfaces in three-space, 79.

Gross, Leonard. On the automorphism group of the bounded algebra of a real Hilbert space, 492; Integration and nonlinear transformations in Hilbert space, 603.

Grünbaum, B. Intersections of cells in normed spaces, 817.

Guy, W. T., Jr. Approximations for the inverse Laplace transform, 319; Operational solution of certain integral equations, 589.

Habetler, G. J. See Martino, M. A.

Habetler, G. J. and Martino, M. A. Concerning the time-independent multigroup diffusion equations, 492.

Haimo, Franklin. Stability groups as splitting extensions. Preliminary report, 74.

Halberg, C. J. A., Jr. and Kramer, V. A. Concerning a theorem of Gelfand and Levitan. Preliminary report, 197.

Halbert, C. J. A., Jr., Levin, Eugene and Straus, E. G. On contiguous congruent sets in Euclidean space, 197.

Hale, J. K. Criteria for orbital stability, 685.

Hall, D. W. See Vanderslice, B. R.

Hall, Marshall, Jr. See Swift, J. D.

Halperin, Israel. See Amemiya, Ichiro.

Halpern, Edward. On the structure of hyperalgebras, 59; Twisted polynomial hyperalgebras, 334; Hopf algebras of class 1, 578; On the cohomology of a space on which an H-space operates, 791.

Hamilton, O. H. Fixed points for special transformations of plane continua, 240.

Hammer, P. C. Functions of bounded variation I, 191; Generalized continuity, 846.

Hamstrom, M.-E. Regular mappings whose inverses are 3-cells, 797.

Hano, Jun-ichi. On complex homogeneous spaces of compact Lie groups, 356.

Harary, Frank. The number of k-chromatic graphs, 159; On the group of the composition of two graphs, 205; On the number of dissimilar graphs between a given graph-subgraph pair, 364; An elementary theorem on graphs, 783; A graph theoretic method for the reduction of a matrix, 789.

Hardell, W. J. Left associated elements in a linear algebra. Preliminary report, 191.

Hardy, F. L. See Evans, Trevor.

Harris, T. E. On random connections, 163.

Harrison, D. K. Some notes on torsion-free Abelian groups, 28.

Harrold, O. G., Jr. A sufficient condition that a monotone image of the three-sphere be a topological three-sphere, 456; Euclidean domains with uniformly abelian local fundamental groups, III, 686.

Hart, W. L. and Motzkin, T. S. Proof of the fundamental theorem on implicit functions by composite gradient corrections, 207.

Hartman, Philip and Sacksteder, Richard. On maximun principles for nonhyperbolic partial differential operators, 98.

Hartmanis, Juris. Algebra of linear sequential coding networks, 181.

Haynsworth, E. V. Generalizations of Brauer theorems on stochastic matrices, 607.

Heaps, H. S. Asymptotic solutions of the equation $y^{IV} + 2x^{-1}y^{III} + y = 0$, 502.

Hedlund, G. A. See Fort, M. K., Jr.

Heerema, Nickolas. Note on complete discrete valuation rings, 80

Heider, L. J. A special class of Boolean algebras, 94; Compactifications of dimension zero, 695.

Helgason, Sigurdur. On a theorem of Sidon, 47; Differential operators on homogeneous Riemannian spaces, 47; Mean value theorems on symmetric spaces, 48; Wave equations on homogeneous spaces, 835.
Heller, Robert, Jr. Some convergence theorems for continued fractions, 682.
Helmberg, G. M. Generating sets of elements in compact groups, 179; A theorem on equidistribution in compact groups. II, 813.
Helson, Henry and Lowdenslager, D. B. Prediction theory for vector valued processes, 325.
Henkin, Leon. Weighted sums of squares, 65.
Henrici, Peter. A unified theory of propagated error and numerical stability for some methods of integrating ordinary differential equations, 55; Error and numerical stability of two three-step methods for integrating $y' = f(x,y)$, 55; The numerical integration of certain ordinary differential equations with missing derivatives. Preliminary report, 56.
Henriksen, Melvin. The $a^{n(a)} = a$ theorem for semirings with commutative addition, 311; Some remarks on a paper of Aronszajn and Panitchpakdi, 320; Ideals in semirings with commutative addition, 321; Multiplicative summability methods and the Stone-Čech compactification, 823.

Hermann, Robert. Obstruction theory in fibre spaces I, 60; Loop spaces of homogeneous spaces, 61.
Herstein, I. N. On a question by Kaplansky in ring theory, 337.
Herstein, I. N. and Ruchte, M. F. Semi-automorphisms of groups, 175.
Heuer, G. A. Algebraic extensions of Banach algebras, 458.
Hewitt, Edwin and Zuckerman, H. S. Some theorems on lacunary Fourier series, with extensions to compact groups, 824.
Heyneman, Robert. Duality in general ergodic theory, 97.
Hildebrandt, T. H. Systems of linear differentio-Stieltjes-integral equations, 310.
Hilton, P. J. and Ledermann, Walter. Homology and ringoids. I, 25; Homology and ringoids. II, 25.
Hilzman, John. Application of the Fréchet differential to the approximate solution of the Volterra integral equation, 375.
Hirsch, M. W. On immersions and regular homotopy of differentiable manifolds, 62; Criteria for immersibility of manifolds, 217.
Hirschman, I. I., Jr. On multiplier transforms. Preliminary report, 152; Capacity in ultraspherical algebras. Preliminary report, 152.
——See Devinatz, Allen
Hiz, Henry. Extendable sentential calculus, 34.
Hodges, J. H. Some determinantal equations related to representations by forms over a finite field, 783; Some exponential sums for matrices over a finite field. Preliminary report, 807.
Hoffman, Kenneth and Singer, I. M. On some Banach algebras of Wermer, 231.
Holladay, J. C. On games with multiple moves, 65.
Holmann, Harald. Complex manifolds with a group of holomorphic automorphisms, 49.
Homann, F. A. Projective geometrical aspects of Laguerre-Forsyth canonical form, 36; On the osculating spaces to analytic projective curves, 236.
Horn, Alfred and Steinberg, Robert. Eigenvalues of the unitary part of a matrix, 353.
Horne, J. G. Rings with effective 0-ideal structure, 592.
Horstman, W. G. See Fort, M. K., Jr.
Horwitz, L. P. and Goldstine, H. H. A procedure for the diagonalization of normal matrices, 29.
Householder, A. S. A class of methods for matrix inversion, 164; On minimal norms, 206; Unitary triangularization of a matrix, 578.
Howard, L. N. Hydrodynamic stability of a jet in two dimensional viscous flow, 64; On hydrodynamic stability of a wake, 229.
Hsiung, C. C. Curvature and Betti numbers of compact Kaehler manifolds with boundary, 185
——See Feeman, G. F.
Hu, S. T. On continuous associative multiplications in arcwise connected compact Hausdorff spaces, 220.
Hughes, D. R. On the H_p-subgroup of a group. Preliminary report, 206.
Hummel, J. A. A variational method for starlike functions, 24; Extremal problems for starlike functions, 813.
Hunter, R. P. Compact connected topo-

logical semigroups, 80; Certain one dimensional semigroups, 690.
Hunzeker, H. L. The torsion of a circular shaft with an ellipsoidal cavity, 311.
Hutcherson, W. R. A note on a surface of order seventeen, 468.

Isaacs, Rufus. Search games with immobile hiders, 70.
Isaacson, Eugene. See Fife, Paul.
Ives, R. T. Uniform shrinkability in topological vector spaces, 47.

James, A. T. Zonal functions of the representation of the linear group in the space of polynomials of a symmetric matrix. Preliminary report, 688.
James, R. C. Banach spaces with a specified number of separable conjugate spaces, 680.
Jenkins, H. B. Dual variational problems, 824.
Jenkins, T. R. and McKnight, J. D., Jr. Ideals in rings of continuous functions, 836.
Jerison, Meyer. See Gillman, Leonard.
Johnson, D. G. A structure theory for a class of lattice-ordered rings. I. Preliminary report, 813.
Johnson, Diane and Robinson, G. de B. The reduction of an ordinary representation of S_n into its modular components for $p \leq n$, 831.
Johnson, Guy, Jr. A bound for the mass distribution of a function which has harmonic support, 310; Limit-convergence of function sequences, 847.
Jones, B. W. and Chowla, Sarvadaman. Note on perfect difference sets, 24.
Jones, F. B. R. L. Moore's Axiom 1' and metrization, 86; A remark on n-point aposyndesis and embedding a cartesian product in the plane, 86; Moore spaces and uniform structure, 152; On the existence of weak cut points in plane continua, 371; Another cutpoint theorem for plane continua, 592.
Jones, F. B. and Young, G. S. Product spaces in n-manifolds, 602.
Jonsson, Bjarni. Homegeneous universal relational systems, 776.
Jurkat, W. B. On semigroups of positive matrices. II, 221; On semi-groups of positive matrices. I, 311; Semi-groups of Toeplitz matrices, 847.

Kaiman, J. A. Complemented lattices in universal algebra, 37.
Kanwal, R. P. Absorption of sound waves in a uniform stream, 474; Acoustic streaming past a sphere in a viscous, compressible and heat conducting fluid. Preliminary report, 801.
Kaplan, Samuel. The second dual of the space of continuous functions. II, 89.
Kaplan, Wilfred. On a theorem of A. Huber, 497.
Karp, C. R. Formalizations of propositional languages with wffs of infinite length. Preliminary report, 172; Formalizations of functional languages with wffs of infinite length. Preliminary report, 172; Formalisms for P_α, $F_{\alpha\beta}$ and α-complete Boolean algebras. Preliminary report, 173; Split semantic models, 679.
Kassimatis, Costas. Functions which have generalized Riemann derivatives, 237.
Keisler, H. J. The theory of models with generalized atomic formulas. I, 676; The theory of models with generalized atomic formulas. II, 776.
Keisler, J. E. A T-like plane continuum which is not a quasi-complex, 482; On generalized Vietoris complexes and nerve complexes. Preliminary report, 825.
Kellogg, Bruce. Hyperbolic equations with multiple characteristics, 326.
Kelly, L. M. Metric curvature in linear spaces, 93.
Kelly, Paul and Straus, E. G. On the projective centers of convex curves, 794.
Kelman, R. B. On the asymptotic behaviour of a differential-functional equation, 836.
Kemeny, J. G. A probability limit theorem requiring no moments, 596.
Kemp, R. R. D. A class of singular non-self-adjoint differential operators, 505.
Kemperman, J. H. B. On the asymptotic distribution of a sequence in a compact group, 192.
Kendall, D. G. Compactness criteria, and one-parameter semigroups of transformations, 158.
Kenyon, Hewitt and Morse, A. P. Runs, 599.
Killgrove, R. See Swift, J. D.
Kimbro, G. M. See Salzer, H. E.
Kimme, E. G. Note on the convergence of sequences of stochastic processes, 479.
Kimura, Naoki. On some special idempotent semigroups (I), 51; Total orderings

on a semilattice, 312; On some special idempotent semigroups (III), 321; Existence theorems on a multiplicative system. II, 493; A theorem on the semigroup of stochastic matrices, 777.
Kimura, Naoki and Yamada, Miyuki. On some special idempotent semigroups (II), 87.
Kincaid, W. M. Convergence of a two-point method for the numerical solution of sets of simultaneous equations, 214.
Kist, J. E. Locally o-convex spaces. III, 468; Invariant positive linear functionals, 801.
Klamkin, M. S. On the solution of a nonlinear integral equation arising in a study of advection fogs, 182; On the steady-state solution of a heat conduction problem with Stephan-Boltzmann radiation losses, 242; One-dimensional heat flow with space-varying thermal properties, 814.
——See Shepp, L. A.
Klamkin, M. S. and Newman, D. J. On the number of distinct zeros of polynomials, 477.
Klee, V. L., Jr. Extremal structure of convex sets. II, 57; Remarks on the geometry of normed linear spaces, 231.
Klee, V. L., Jr. and Long, R. G. On a method of mapping due to Kadeč and Bernstein, 42.
Kleene, S. C. Extension of an effectively generated class of functions by enumeration, 30.
Klimczak, W. J. The domain of absolute convergence of certain eigenfunction series, 371.
Klingenberg, Wilhelm. A characterization of Hermitian manifolds, 96.
Knopp, M. I. On certain automorphic functions, 801.
Koch, R. J. Note on weak cutpoints in clans, 78; Arcs in partially ordered spaces, 687; Ordered semigroups in partially ordered semigroups, 784.
Koch, R. J. and Krule, I. S. Concerning the weak cutpoint ordering, 784.
Kochen, S. B. Filtration systems. I, 605; Filtration systems. II, 671.
Kohlbecker, E. E. Asymptotic properties of partitions, 63; Asymptotic properties of the sum of the divisors of an integer, 227.
Kohls, C. W. Prime ideals in rings of continuous functions. Preliminary report, 44.
Kohn, J. J. See Adler, A. W.
Kokoris, L. A. Nodal flexible associative-admissible algebras, 90; Simple nodal noncommutative Jordan algebras, 158; On nilstable algebras, 209.
Kooharian, Anthony. See Fleischer, Isidore.
Koppelman, Walter. On the spectral theory of singular integral operators, 599.
Korevaar, Jacob. Pansions, Fourier transforms and boundary value problems, 232.
Kozin, Frank. On probability averages of functions, 184; On the quadratic detector. I, 579.
Krabbe, G. L. Normal operators on a Banach space, 185; A space of multipliers of Fourier integrals, 807.
Kramer, H. P. Best finite linear approximation to a second order stochastic process, 201; Symmetrizable Markov matrices, 230.
Kramer, V. A. See Halberg, C. J. A., Jr.
Kravetz, Saul. On the local geometry of Teichmueller spaces. I. Preliminary report, 50.
Kreider, D. L. On sets in both two-function-quantifier forms, 364.
Kreyszig, E. O. A. On partial differential equations of the fourth order, 75; Properties of a class of oscillating systems having a limit cycle, 157; Singularities of solutions of partial differential equations in three real variables, 463; On the eigenvalues of Hermitian matrices, 579; On harmonic functions of three variables, 663; A numerical method for solving the torsion problem of a bar, 777.
Kreyszig, E. O. A. and Todd, John. The radius of univalence of Bessel functions. I, 664.
Krickeberg, Klaus. Semimartingales going down, 360.
Krule, I. S. See Cohen, Haskell; Koch, R. J.
v. Krzywoblocki, M. Z. On the superposability and self-superposability of certain partial differential equations, 162.
Kunze, R. A. and Stein, E. M. Some unusual aspects of harmonic analysis on the group of 2×2 real unimodular matrices, 250.
Kwak, Nosup. Embedding compact metric spaces in cubes, 82.

Kwun, K. W. A sufficient condition that an upper semicontinuous decomposition of S^3 yield a topological 3-sphere, 579; Characterization of the n-sphere, 595.

Kyner, W. T. On the computation of a cut, 74; An existence and uniqueness theorem for a nonlinear Stefan problem, 251.

Kyner, W. T. and Lees, Milton. The structure of electrical networks, 678.

Laasonen, Pentti. A Ritz method for simultaneous determination of several eigenvalues and eigenvectors of a matrix, 201; On the difference methods in solving inhomogeneous elliptic differential equations, 304.

Laha, R. G. On a factorization theorem in the theory of analytic characteristic functions, 183; On a class of distribution functions where the quotient follows the Cauchy law, 799.

Lambek, Joachim. See Brainerd, B.; Findlay, G. D.

Landau, H. J. On canonical conformal maps of multiply connected domains. I, 153.

——See Walsh, J. L.

Lane, R. E. Continued fraction expansions for functions, 150.

Lange, L. H. A non-Euclidean analogue to a theorem of H. Milloux and its relationship to a theorem of W. Seidel, 847.

Langenhop, C. E. Mean functions of almost periodic functions on a group. Preliminary report, 219.

Langer, R. E. Formal solutions and a related equation for a class of fourth order differential equations of a hydrodynamic type, 207.

Lanzane, Paolo. Projective-geodesic mapping of surfaces. Preliminary report, 825.

Lashof, R. K. and Smale, Stephen. On the self-intersection of an immersed manifold, 458.

Laurent, A. G. On the "generalized correlation" between vectors. Preliminary report, 29.

Lax, P. D. The eigenvalues as monotonic matrix functions, 30.

Leader, Solomon. On clusters in proximity spaces, 600.

Ledermann, Walter. See Hilton, P. J.

Ledley, R. S. The functional equations in finite Boolean algebras, 46.

Lees, Milton. Approximate solutions of parabolic equations, 232.

——See Kyner, W. T.

Lees, Milton and Radok, J. R. M. Explicit difference methods with high order accuracy for parabolic equations, 58.

deLeeuw, Karel. See Rudin, Walter.

Leger, G. F. and Tôgô, Shigeaki. On characteristically nilpotent Lie algebras, 837.

Lehman, Alfred. A remark on wye-delta transformations, 191.

Lehmer, D. H. Compositions of basic divisor functions, 68; On the exact number of primes less than a given limit, 357.

Lehmer, Emma. Conditions for quartic residuacity, 75.

Lehner, G. R. Extending homeomorphisms on the pseudo-arc, 212.

Lehner, Joseph. On modular forms of dimension -2, 580; The Fourier coefficients of automorphic forms on horocyclic groups. II, 694.

Leipnik, Roy. Linear integral equations for the generating functions of a class of functionals of Markov processes, 377.

Lemke, C. E. See Charnes, Abraham.

Lester, Anne. Some semigroups on the two-cell, 484.

Levin, Eugene. See Halberg, C. J. A., Jr.

Levine, H. I. An order function for analytic mappings of complex manifolds into projective space. Preliminary report, 38.

Levine, Jack. Note on the number of pairs of nonintersecting permutations, 691.

Li, Ta. On a new class of integral transforms, 790.

Lima, E. L. Stable Postnikov invariants, 215.

Lissner, D. B. 2×2 matrices of trace 0, 664.

Littman, Walter. Finite difference methods for elliptic equations of arbitrary order, 372; A strong maximum principle for weakly L-subharmonic functions, 837.

Livingston, A. E. and Lorch, Lee. The zeros of some sine-integral-like functions, 377.

Lloyd, S. P. A sampling theorem for stationary (wide sense) stochastic processes, 158; Stochastic processes with analytic covariance, 789.

Loeb, H. L. Rational fraction approxima-

tions at discrete points, 87; Algorithm for Chebycheff approximations using the ratio of linear forms. Preliminary report, 837.
Long, R. G. See Klee, V. L., Jr.
Lorch, Lee. On the Lebesgue constants for Laplace series, 212; The Lebesgue constants for the Borel and Euler summation of developments in Legendre polynomials, 312; The Lebesgue constants for Jacobi series, 817.
——See Livingston, A. E.
Lorch, Lee and Szego, Peter. A singular integral whose kernel involves a Bessel function. II, 602.
Lorentz, G. G. See Erdös, Paul.
Losey, G. O. Ideals of index 1 in group rings, 94; The dimension subgroups of a group, 168; On the decompositions of a group algebra into a sum of a field and a nil ideal, 580.
Loud, W. S. Periodic solutions of $x'' + cx' + g(x) = \epsilon e(t)$. II, 91; Periodic solutions of a perturbed autonomous system, 186; Periodic solutions of a perturbed autonomous system. II, 493.
Lowdenslager, D. B. See Helson, Henry.
Lumer, Günter. Numerical range and Banach space geometry. Preliminary report, 212.
Lumer, Günter and Rosenblum, Marvin. Linear operator equations, 243.
Lyndon, R. C. A semantic Herbrand-Gentzen lemma, 464; Direct products of algebras, 848.

McArthur, C. W. On totally bounded sets in sequence spaces, 326.
McAuley, L. F. Concerning an unnecessary condition in certain upper semicontinuous decompositions of E^3 into itself, 217; Some upper semi-continuous decompositions of Euclidean 3-space into itself, 327; Local cyclic connectedness. I. Cyclic subelements of cyclic elements, 698.
McCarthy, P. J. Note on indefinite ternary genera of more than one class, 180.
McCauley, A. N. On the existence of real eigenvalues for a non-self-adjoint boundary-value problem, 38.
McCulloh, L. R. Integral bases in Kummer extensions of algebraic number fields. Preliminary report, 808.
McDougle, P. F. A theorem on quasi-compact mappings, 33; Mapping and space relations; 344; Semi-compact mappings, 345.
McFarland, J. E. An iterative solution of the quadratic equation in Banach space, 381.
McGarvey, D. C. Operators commuting with translation by one, 508.
McKelvey, R. W. Singular second order differential boundary problems of Fourier type, 838.
McKibben, J. J. Singular integrals in two dimensions, 184.
McKinney, R. L. Positive bases for linear spaces, 678.
McKnight, J. D., Jr. See Bagley, R. W; Jenkins, T. R.
McLaughlin, J. E. Prime power representations of finite simple Lie groups, 581.
——See Griffin, J. S., Jr.
McMinn, T. J. On the surface line segments of a convex set, 332.
McShane, E. J. A canonical form for antiderivatives, 456.
McWilliams, R. D. Closed extensions of unbounded linear transformations, 600; Projections of separable spaces of bounded sequences onto the space of convergent sequences, 838.
MacCamy, R. C. The principle of Babinet, 41.
MacColl, L. A. Appell's equations in relativistic particle dynamics, 794.
MacLane, G. R. On the growth of entire functions, 301.
MacNerney, J. S. Generalized chain sequences. Preliminary report, 190.
Mahowald, M. E. A summability theorem in countable toral groups, 221; On discontinuous functions, 478; Summability in compact abelian groups, 798.
Malm, D. G. On the cohomology ring of a sphere bundle, 460.
Mandan, S. R. and Schuster, Seymour. Some properties of certain pencils of conics, 818.
Mann, H. B. The algebra of a linear hypothesis, 808.
Mann, L. N. On a theorem of Floyd. Preliminary report, 593.
Marcus, M. D. and Moyls, B. N. Linear transformations on algebras of matrices, I, 805.
Marcus, M. D. and Newman, Morris. Permanents of doubly stochastic matrices, 313.
Marcus, M. D. and Ree, Rimhak. Diagonals of doubly stochastic matrices, 376.

Marden, Morris. A discontinuous eigenfunction problem. Preliminary report, 347; On the zeros of infrapolynomials for partly arbitrary point sets, 581.
Mardešić, Sibe. Comparison of singular and Čech homology in locally connected spaces, 210; On a problem concerning local connectedness in the sense of singular homology, 684; Dimension and inverse limits of compact spaces, 785.
Margulies, Gabriel. The differential invariants of local regular surface mappings, 854.
Markus, L. F. Structurally stable differential equations, 826.
Marshall, A. W. A multivariate generalization of Chebyshev's inequality. Preliminary report, 354.
Martin, M. H. Linear and nonlinear boundary problems, 480.
Martinek, Johann. See de Beaumont, Henry; Yeh, G. C.K.
Martino, M. A. An extension of a spectral theorem of M. A. Naĭmark, 344; A resolution of the identity for certain unbounded linear operators, 697.
——See Habetler, G. J.
Martino, M. A. and Habetler, G. J. On the multigroup diffusion model, 215.
Maserick, P. H. Set functions and translation half-rings in the plane, 178.
Massera, J. L. and Schäffer, J. J. Linear differential equations and functional analysis. V, 665; Linear differential equations and functional analysis. VI, 666.
Massey, W. S. On the cohomology ring of sphere bundles, 26; On the non-imbeddability of certain manifolds in Euclidean space, 27; On the second obstruction in 2-sphere bundles, 27.
Matlis, Eben. Injective modules, 204.
Matsuno, Shozo. See Greenspan, Donald.
Maxfield, J. E. See Flanagan, Czerna.
Maxfield, J. E. and Flanagan, Czerna. Estimates of the roots of certain polynomials, 466.
Maxfield, J. E. and Selfridge, R. G. Lin's method applied to translated polynomials, 81.
Maybee, J. S. See Fulks, W. B.
Mayer-Kalkschmidt, Jörg. Conditional inclusion of summability methods, 317.
Meisters, G. H. On almost periodic solutions of a class of differential equations, 218; On the almost periodicity of the integral of an almost periodic function, 683; Almost periodic motions in uniform spaces, 802.
Melzak, Z. A. Limit sections and universal points of convex surfaces, 484; Entire operators and functional equations, 581.
Mendelson, Elliott. The Axiom of Fundierung and the axiom of choice, 455.
Mendelson, Pinchas. On dynamical systems without improper saddle points. I, 842.
Mesner, D. M. An embedding theorem for 0, 1 matrices, 327.
Meyers, Norman. Expansion of solutions of second order linear elliptic equations about infinity, 335.
Michael, E. A. Cross sections in certain topological groups, 66; Yet another characterization of paracompact spaces, 202; Convex structures and continuous selections, 359.
Mihalek, R. J. Modularity relations in lattices. Preliminary report, 349; Modularity relations in lattices, 826.
Miles, E. P., Jr. Expansion of an arbitrary polynomial in terms of a basic set of polyharmonic polynomials, 162; The Cauchy problem for the iterated wave equation, 245; An approach to certain reflection principles, 498.
Miller, D. W. An imbedding theorem for a class of nets. Preliminary report, 589.
Mills, H. D. Noncooperative games and nonlinear programs, 808.
Milne, W. E. and Reynolds, R. R. Stability of a numerical solution of differential equations, 231.
Milnes, H.W. and Potts, R. B. Stability criteria for the numerical solution of partial differential equations by boundary contraction, 598.
Milnor, J. W. On the cobordism ring Ω^*, 457.
Minty, G. J. A general algorithmic method for routing and network problems, 682.
Miranker, W. L. and Frisch, H. L. Analysis of the nonlinear Stefan problem, 506.
Mizel, V. J. A boundary layer result for an N-dimensional linear elliptic equation, 51.
Montague, Richard. Incomparable theories, 362.
Moore, C. N. On the frequency of prime

triples, 160.
Mordell, L. J. On the inequality $\sum_{r=1}^{n}(x_r/(x_{r+1}+x_{r+2}))$ > n/2 and some others, 582.
Morel, A. C. See Frayne, T. E.
Morel, A. C., Scott, D. S. and Tarski, Alfred. Reduced products and the compactness theorem, 674.
Morrey, C. B., Jr. The analytic embedding of abstract real-analytic manifolds, 73; Elliptic differential equations with Hölder-continuous coefficients, 466.
Morse, A. P. See Kenyon, Hewitt.
Moser, Leo. On the minimal overlap problem of P. Erdös, 839.
Moser, William. An abstract definition for the Mathieu group of degree twelve, 795.
Mostert, P. S. On a compact Lie group acting on a sphere, 498.
Mostert, P. S. and Shields, A. L. One-parameter semigroups in a semigroup, 81.
Motzkin, T. S. See Hart, W. L.
Moyls, B. N. See Marcus, M. D.
Munkres, J. R. Differentiable isotopies on the two-sphere, 582.

Nash, J. F. The decrease of fundamental solutions of parabolic equations with distance from the initial singularity, 44.

Nerode, Anil. Linear automaton transformations, 186; Remarks on isolic arithmetic, 216; Some Stone spaces and recursion theory. I, 346; Remarks on isolic arithmetic. II, 677.
Nerode, Anil and Shank, H. Theorems of Kirchhoff type, 183.
Neuberger, J. W. Concerning boundary value problems, 583; On the existence of certain Green's functions, 814.
Neugebauer, C. J. A decomposition of a Peano space, 194; Local A-sets and retractions onto B-sets, 194; A fine-cyclic additivity theorem for a functional, 195.
—— See Cesari, Lamberto.
Newman, D. J. Partitioning of areas by straight lines, 510.
—— See Klamkin, M. S.
Newman, Morris. Further identities and congruences for the coefficients of modular forms, 476.
—— See Marcus, Marvin.
Newman, Morris and Reiner, Irving. Inclusion theorems for congruence subgroups, 54.
Nijenhuis, Albert. A generalization of Walker's differential operator, 327; Hyperconvexity in Riemannian manifolds, 848.
Nitsche, J. C. C. Isolated singularities of solutions of regular variational problems, 180; On Riemann's problem of function theory, 839.
—— See Nitsche, Joachim.
Nitsche, Joachim. Simple error bounds for eigenvalues and eigenvectors of hermitian matrices, 777.
Nitsche, Joachim and Nitsche, J. C. C. A criterion for the existence of nonlinear entire solutions of elliptic equations, 46.
Nohel, J. A. Stability and instability in reactor dynamics. Preliminary report, 174; A nonlinear delay-differential equation, 826.
Norman, R. Z. and Rabin, M. O. An algorithm for a minimum cover of a graph, 36.
Norton, D. A. Conjugate invariants. Preliminary report, 506.
Nussbaum, A. E. Integral representation of semigroups of unbounded self-adjoint operators, 95.

O'Keefe, E. S. A theorem on independence of universal algebras, 486.
Oliphant, T. A. See Baker, G. A., Jr.
Oliver, H. W. Spectra in a real Banach algebra, 340.
Ong, R. S. B. See Coburn, Nathaniel.
Orey, Steven. Recurrent Markov chains II, 849.
Osborn, J. M. A new class of neofields, 849.
Osserman, Robert. Relations between mean curvature and conformal type, 50; Minimal surfaces with bounded normal direction, 681.
Ostrom, T. G. and Wagner, A. Transitive affine planes and doubly transitive projective planes, 179.

Padmavally, K. A mixed boundary value problem, 174.
Paige, L. J. See Blattner, J. W.
Paine, R. B. Fixed points of the circle group acting on a cohomology n-sphere. Preliminary report, 210.
Palais, R. S. Extending germs of diffeomorphisms, 32; Natural operations on differential forms, 33.
Palermo, F. P. A universal coefficient

system for cohomology rings, 827.
Parker, E. T. Construction of some sets of pairwise orthogonal latin squares. Preliminary report, 815.
Parter, S. V. On the eigenvalues and eigenvectors of a class of matrices, 696.
Parzen, Emanuel. Statistical inference on time series by Hilbert space methods, I, 839.
Payne, L. E. See Pell, W. H.
Payne, L. E. and Weinberger, H. F. Bounds in the exterior Neumann problem, 173.
Pearl, Martin. On Cayley's parameterization. II, 166.
Pell, W. H. and Payne, L. E. Application of generalized axially symmetric potential theory to Stokes flow problems, 468.
Pellicciaro, E. J. On the compatibility of systems of linear differential equations, 690.
Penico, A. J. Guided monochromatic waves in an anisotropic medium, 591.
Penzlin, F. See Dolph, C. L.
Perlin, I. E. Function approximation by broken-line functions, 148.
Petersen, G. M. Bounded sequences summed by matrices, 214; L-regular matrices, 827.
Peterson, F. P. Some remarks on complex n-plane bundles, 223.
Peterson, F. P. and Thomas, Emery. Functional operations associated with nonadditive cohomology operations, 328.
Phelps, R. R. Subreflexive normed linear spaces, 165; Some subreflexive Banach spaces, 353.
Pierce, R. S. Representation theorems for certain Boolean algebras, 170; Translation lattices. I, 374; Translation lattices. II, 471.
Pierce, R. S. and Christensen, D. J. Free products of alpha-distributive Boolean algebras, 486.
Pimbley, G. H. A proof of existence and uniqueness for the solution of a discrete nonlinear boundary value problem, 365; Solution of the initial value problem for the linearized multi-velocity transport equation with a slab geometry, 366.
Pless, Vera. Quotient rings of continuous transformation rings, 188; On the continuous transformation ring of biorthogonal bases spaces, 188.
Plunkett, R. L. A topological proof of the continuity of the derivative of a function of a complex variable, 682.
Pollak, Barth. Concerning a hermitian equation in a quaternion algebra, 313; Quadratic forms over rings, 840.
Pollard, Harry. See Blackman, Jerome.
Pollard, Harry and Widom, Harold. Inversion of a class of convolutions, 213.
Porcelli, Pasquale. On the reduction of weak convergence, 322; On the equivalence of weak and norm convergence in the space of function of bounded variation, 481; Weak convergence and weak compactness in the space of functions of bounded variation, 815.
Portmann, W. O. A derivative for Hausdorff-analytic functions, 467.
Posner, E. C. Algebraic sets and finitely generated semigroups, 216; Two theorems on (C, 1) summability, 345; An alternate characterization of the lower nil radical, 583.
Potts, R. B. See Milnes, H. W.
Poulsen, E. T. On the Dirichlet problem for a nonuniformly elliptic differential operator, 355.
Preisendorfer, R. W. Principles of invariance and the planetary hydrosphere problem, 70; Asymptotic radiance distributions in eventually separable media, 506.
Prenowitz, Walter. A theory of incidence geometries, 583.
Preston, G. B. An embedding theorem for semigroups, 83; On a semigroup of order isomorphisms determined by an ordered semigroup, 356.
Preston, G. B. and Bleicher, M. N. Abstract linear dependence relations, 507.
Prosser, R. T. Representations of *-algebras, 603; On the existence of a scattering operator, 664.
Puckette, S. E. Semigroups and unbounded difference operators, 486.
Punga, Valdemars. On the Monte Carlo solution of simultaneous linear equations, 49; Simplified simplex method applied to the solution of simultaneous linear equations, 483.
Puppe, Dieter. A theorem on semisimplicial monoid complexes, 216.
Putnam, Hilary and Davis, Martin. Uniqueness ordinals and extended R-systems, 607.
Putzer, E. J. On an Nth order system of differential equations, 489.

Rabin, M. O. See Norman, R. Z.
Radlow, James. Diffraction by a quarter-plane, 23; Diffraction by a dielectric wedge. Preliminary report, 202.
Radok, J. R. M. The partial differential equation analogue of an infinite system of ordinary differential equations, 75; Method of functional extrapolation for the numerical integration of ordinary differential equations, 354.
——See Lees, Milton.
Rainich, G. Y. Inversive geometry, complex numbers and proportionality, 698.
Rapaport, E. S. Finite groups. Coset decompositions, 463; Finite groups. Graphs, 463; Groups with L-polynomials, 604.
Raymond, F. A. Cech homology from a chain complex, 203; A new proof for duality theorems in generalized manifolds, 298; On factorization of generalized manifolds, 298.
Ree, Rimhak. See Marcus, M. D.
Reich, Edgar. Solution of the Takács integrodifferential equation, 69.
Reid, W. P. Heat flow in a composite solid, 493.
Reid, W. T. A Prüfer transformation for differential systems, 233.
Reiner, Irving. On the class number of representations of an order, 584.
——See Newman, Morris.
Reinhart, B. L. Foliated manifolds with bundle-like metric. I, 171; Foliated manifolds with bundle-like metric. II, 171; Line element fields on the torus. Preliminary report, 476; Harmonic integrals on foliated manifolds, 596.
Renggli, Heinz. Note on extension of quasiconformal mappings, 148; An inequality for logarithmic capacities, 299.
Reynolds, R. R. See Milne, W. E.
Reynolds, W. F. Blocks with normal defect group, 480.
Reza, F. M. On the Schlicht behaviour of certain impedance functions, 83.
Rhoades, B. E. Total comparison between two Cesàro matrices, 828.
Richardson, R. W., Jr. Actions of the groups S0(3) and Sp(1) on the four-sphere, 509; Actions of the groups S0(3) and Sp(1) on the five-sphere, 584.
Riggle, E. C. An alternating gradient method for the solution of linear systems, 380.
Rim, D. S. Modules over finite groups and cohomology, 481.
Rinehart, R. F. Skew matrices as square roots, 809.
Riney, T. D. Coefficients in certain asymptotic factorial expansions of the second kind, 475.
Riordan, W. J. On the interpolation of operations, 590.
Rivlin, T. J. On the maximum modulus of polynomials, 809.
Roberts, J. B. Polynomial identities. I, 792.
Roberts, J. H. Contractibility of spheres in spaces of homeomorphisms, 213.
Robertson, M. S. Schlicht Dirichlet series, 190; Cesàro partial sums of harmonic series expansions, 499.
Robinson, D. W. A note on diagonable matrix polynomials, 319.
Robinson, G. de B. See Johnson, Diane.
Robinson, L. B. Short proof of a theorem of Riquier, 38; Solution of a problem proposed by I. M. Sheffer, 601.
Robinson, S. M. On the existence of the fundamental solution of the parabolic equation, 686.
Rogers, G. S. A note on order statistics and stochastic independence, 487.
Room, T. G. Spinors and quaternions, 243.
Rosen, J. B. Fourier series approximation to nonlinear parabolic boundary value problems, 223.
Rosen, R. H. Fixed points for multivalued functions on snake-like continua, 350.
Rosenberg, Alex and Zelinsky, Daniel. Finiteness of the injective hull, 169.
Rosenblatt, Murray. See Burke, C. J.
Rosenbloom, P. C. Probability, entire functions, and statistical mechanics, 591; Boundary value problems for hyperbolic equations, 835.
Rosenbloom, P. C. and Widder, D. V. Representation of solutions of the heat equation, 350.
Rosenblum, Marvin. On the Hilbert matrix. II, 245.
——See Lumer, Günter.
Roth, W. E. Determinants of certain composite matrices, 335; The matric equation $X^2 - Y^2 = A$, 335.
Royster, W. C. See Cowling, V. F.
Rubel, L. A. Extreme limits and Tauberian theorems, 35; A limit theorem for real functions, 314.
Rubin, Herman. On a problem of Kurepa concerning the axiom of choice, 378;

Convergence of distributions on function spaces, 357.
Ruchte, M. F. See Herstein, I. N.
Rudin, Walter. On the structure of maximum modulus algebras, 336; Fourier-Stieltjes transforms and weak almost periodic functions, 340; Functions which operate on Fourier-Stieltjes transforms, 849.
Rudin, Walter and deLeeuw, Karel. The extreme points of the unit ball in H_1, 177.
Russell, H. G. See Walsh, J. L.
Rutledge, W. A. See Green, Simon.
Ryser, H. J. Inequalities of compound and induced matrices with applications to combinatorial analysis, 35.

Sabidussi, G. O. The structure of graphs, 54; On a class of fixed-point-free graphs, 87; Graphs with transitively acting groups, 233; On the set of graphs with a finite number of given blocks, 593.
Sacksteder, Richard. See Hartman, Philip.
Salzer, H. E. A mixed-type best approximation problem, 372.
Salzer, H. E. and Kimbro, G. M. Tables for bivariate osculatory interpolation over a Cartesian grid, 69.
San Soucie, R. L. Sequences with property G, 319.
Sarafyan, Diran. An infinite set or class of formulas for the computation of roots, 78; Existence of singular points for a well known class of nonhomogeneous differential equations, 314; Existence of singular points for a well known class of nonhomogeneous differential equations, 499.
Scalora, F. S. Abstract martingale convergence theorems, 494.
Scanlon, J. C. Poincaré's perturbation method and topological degree, 241.
Schaefer, H. H. On the Fredholm alternative in locally convex spaces, 376.
Schafer, R. D. Nodal noncommutative Jordan algebras and simple Lie algebras of characteristic p, 828.
Schäffer, J. J. Function spaces with translations, 665.
——See Massera, J. L.
Schechter, Samuel. On the inversion of certain matrices, 792.
Schechter, Martin. On estimating partial differential operators. III, 28; On the coerciveness of linear second order partial differential operators, 184; General boundary value problems for elliptic partial differential equations. I, 666; General boundary value problems for elliptic partial differential equations. II, 667; General boundary value problems for elliptic partial differential equations. III, 788.
Schenkman, E. V. The similarity between the properties of ideals in commutative rings and the properties of normal subgroups of groups. Preliminary report, 189; On the norm of a group, 697.
Scherk, Peter. On regular bilinear forms, 809.
Schild, Albert. On a class of star shaped mappings, 314; On the bound of convexity of a class of star shaped mappings. Preliminary report, 499.
Schoenberg, I. J. Some polynomial extremal problems and mechanical quadratures, 608.
Schreiber, Morris. Representation of operators, 350.
Schroer, D. E. Unified formulation of logical systems, 251; Unified interpretation of logical systems, 460; Combinatory formulation of standard theories, 485.
Schuster, Seymour. Pencils of null systems, 44; Pencils of null systems. II, 239.
——See Mandan, S. R.
Schützenberger, M. P. On a representation of commuting monoids, 31.
Schwartz, P. B. See Evans, Trevor.
Schweizer, Berthold and Sklar, Abe. Statistical metric spaces arising from sets of random variables, 219; On triangle inequalities for statistical metric spaces, 230; On triangle inequalities for statistical metric spaces II, 500; Neighborhoods in statistical metric spaces, 500.
Schwerdtfeger, H. W. E. On the method of minimum deviation, 242; On the groups where cosets are classes, 828.
Scott, D. S. Definability in polynomial rings, 221; On the Löwenheim-Skolem theorem in weak second-order logic, 778.
——See Frayne, T. E.; Morel, A. C.
Scott, D. S. and Tarski, Alfred. Extension principles for algebraically closed fields, 778.
Scott, T. E. See Frayne, T. E.
Segal, Jack. A note on the pseudo arc, 78;

Homogeneity of inverse limit spaces, 687.
Selden, John. A note on topological semirings, 500.
Selfridge, J. L. and Straus, E. G. On the determination of numbers by their sums of a fixed order, 608.
Selfridge, R. G. See Maxfield, J. E.
Seligman, G. B. On the group of invariant automorphisms of a Lie algebra, 779.
Serrin, James. The Harnack inequality for elliptic partial differential equations in more than two independent variables, 52; On solutions of elliptic partial differential equations which are positive near an isolated singular point. I, 53; On solutions of elliptic partial differential equations which are positive near an isolated singular point. II, 53.
Shah, S. M. On meromorphic functions of finite order, 693.
Shah, S. M. and Singh, S. K. Meromorphic functions with maximum defect, 850.
Shank, H. See Nerode, Anil.
Shanks, Daniel. A circular parity switch and applications to number theory, 96.
Shapiro, H. S. Linear extremal problems for polynomials, 248; A power series with small partial sums, 366; Approximation by rational functions, 366; Extremal measures, linear functionals, and Tchebycheff approximation, 597; On functions which satisfy rational recurrence formulae and related classes of real numbers. Preliminary report, 829.
Shapiro, J. M. An identity theorem for the Poisson distribution, 166; Some attraction properties of the Poisson distribution, 829.
Shapiro, V. L. The curl of a vector field, 193; Generalized exterior differentiation, 482.
Shepp, Lawrence. Sets of sequences on which certain generalized limits coincide, 489.
Shepp, Lawrence and Klamkin, M. S. On least square circles and spheres, 368.
Shields, A. L. On complex difference equations, 328.
——See Brown, Leon; Mostert, P. S.
Shinbrot, Marvin. Singularly perturbed integral equations, 26; On the amalgamation of two methods for the solution of integral equations, 29.
Shisha, Oved. Nearest functions, 853.
Shoenfield, J. R. Constructive ordinals in formal systems. Preliminary report, 249; Degrees of decidability of formal theories, 315; The class of recursive functions, 322.
Shützenberger, M. P. Random walk on monoids, 341; Homomorphisms of a monoid onto a group, 341.
Silverman, Edward. A problem of least area, 850.
Silverman, R. J. and Yen, Ti. Characteristic functionals, 315; The Hahn-Banach extension theorem, 316.
Simon, A. B. On the maximality of vanishing algebras, 804
Sinclair, Annette. See Walsh, J. L.
Singer, I. M. See Hoffman, Kenneth.
Singh, S. K. A note on entire functions. Preliminary report, 798.
——See Shah, S. M.
Sion, Maurice. On integration of 1-forms, 667.
Sklar, Abe. See Schweizer, Berthold.
Slepian, Paul. See Weinberg, Louis.
Slotnick, D. L. Asymptotic behavior of solutions of canonical systems near a closed, unstable orbit, 240.
Smale, Stephen. The classification of immersions of spheres in Euclidean spaces, 66.
——See Lashof, R. K.
Small, L. W. A connection between many valued logics and the two-valued system, 238; Relation between recurrent events and an inductive law, 364; On certain aspects of relatable functions, 489.
Smiley, M. F. A remark concerning the commutativity of rings, 167.
Smith, J. W. Stability of quasi-linear difference equations, 22.
Snapper, Ernst. Multiples of divisors, 477; Polynomials associated with divisors, 810.
Snell, J. L. and Williamson, R. E. Representation of martingales, 830.
Sobczyk, Andrew. Pseudo-ergodic random processes. Preliminary report, 83; Discrete random processes. Preliminary report, 299; Stability of two-parameter systems, 687; Homogeneous fibre structures, 850.
Solomon, Louis. The representation of finite groups in algebraic number fields, 840.
Sonner, Johann. On limits of module-systems, 803.

Spector, Clifford. Strongly invariant hierarchies. Preliminary report, 851.
Spencer, D. E. On generalized Newtonian mechanics, 34; On electrodynamics in material media, 469; On the Liénard potentials, 604, On the Lorenz potentials, 630.
Spitzer, F. L. Toeplitz matrices and the absorption problem, 234; An extension of the renewal theorem, 694.
Spitzer, F. L. and Widom, Harold. Inversion of Toeplitz matrices, 163.
Springer, George. Interpolation problems for functions of several complex variables, 471.
Stallings, J. R. Fixed point theorems for connectivity maps, 490; Unknotted curves on 2-manifolds in 3-manifolds, 585; Commutators in free products with amalgamation, 585.
Stampfli, J. G. A spectral resolution for a bounded linear operator some power of which is normal, 830.
Stasheff, J. D. The "space of loops" isomorphism, 252.
Stearns, Richard. Some results on preference patterns. Preliminary report, 348.
Stein, E. M. See Kunze, R. A.
Stein, Paul. On a class of quadratic transformations, 586.
Steinberg, Robert. Variations on a theme of Chevalley, 672.
——See Horn, Alfred.
Stern, T. E. See Friedland, Bernard.
Stern, T. E. and Friedland, Bernard. Application of modular sequential circuits to single error-correcting p-nary codes, 811.
Sternberg, Shlomo. See Brauer, F. G.
Stewart, C. E. On the convergence of sequences of singluar integrals of Cauchy type. Part I, general, 501; On the convergence of sequences of singular integrals of Cauchy type. Part II, polygonal approximations, 586; On the existence of singular integrals of Cauchy type, 587.
Stinespring, W. F. A sufficient condition for a integral operator to have a trace, 252.
Stoll, W. F. Meromorphic maps into complex spaces, 473; About the concept of meromorphic maps, 597; Meromorphic modifications, 841.
Stone, A. H. Metrisability of unions of spaces, 668; Universal spaces for some metrisable uniformities, 779.
Stone, J. J. See Goldman, A. J.
Stoneham, R. G. A study of 60,000 digits of "e". Preliminary report, 64.
Storey, C. R. On the structure of threads. Preliminary report, 507.
Straus, E. G. Sierpiński sets in groups, 66.
——See Cahn, Albert; Halberg, C. J. A., Jr.; Kelly, Paul; Selfridge, J. L.
Stromberg, Karl. Probabilities on a compact group, 793.
Strother, W. L. Orbits in fixed sets, 507.
Stroud, A. H. Functions of bounded variations. II, 198.
Struble, R. A. On singular solutions of certain nonlinear differential equations from the theory of interior ballistics. Preliminary report, 691.
Struik, S. R. To the affine theory of area, 84; Bolzano and the historic significance of his contributions to modern calculus, 299.
Sucheston, Louis. On sequences of events, 853.
Sugai, Iwao. On unequal partitions of integers under certain imposed conditions, 56; Extension of methods used in solving Riccati's nonlinear differential equation, 473.
Supnick, Fred and Cohen, H. J. On the powers of a real number reduced modulo one, 39.
Swanson, C. A. Asymptotic perturbation series for characteristic value problems, 376.
Swift, J. D., Hall, Marshall, Jr. and Killgrove, R. On planes of order nine, 810.
Swingle, P. M. Connected sets of Van Vleck, 86; Connected towers of Wada, 175; Hereditarily indecomposable towers and related sets, 176; Connected sets of Vitali, 176; Widely connected algebras, 501; Lebesgue nonmeasurability and connected locally connected algebras, 587; Continua with sets of indecomposability, 689.
Szego, Peter. See Lorch, Lee.
Taam, C. T. A note on nonlinear oscillations, 39; On compact linear transformations in Banach space, 851.
Taft, E. J. The Whitehead first lemma for alternative algebras, 61.

Takenaga, Roy. Representation of Banach algebras by algebras of matrix-valued functions, 60.
Talacko, J. V. On stochastic linear inequalities, 697.
Tarski, Alfred. Models of universal sentences in predicate logic with infinitely long formulas, 67; Some model-theoretical results concerning weak second-order logic, 673.
——See Frayne, T. E.; Morel, A. C.; Scott, D. S.
Tauskky, Olga. On a matrix theorem of Craig and Hotelling, 72.
Taylor, A. E. The norm of a real linear transformation in Minkowski space, 225.
Taylor, W. C. Numerical solution of algebraic equations, 218.
Thomas, Emery. See Peterson, F. P.
Thompson, J. G. Frobenius groups and a related problem, 695.
Thorp, E. O. Projections onto the subspace of compact operators, 672.
Todd, John. The condition of certain matrices. III, 72.
——See Kreyszig, E. O. A.
Tôgô, Shigeaki. See Leger, G. F.
Trampus, Anthony. Differentiability and analyticity of functions in linear algebras, 802.
Treybig, L. B. Concerning local separability in locally peripherally separable spaces, 587.
Trotter, H. F. On the "product" of semigroups of operators, 604.
Tucker, A. W. Column-dependence relations on matrices, 772.
Tucker, D. H. Absolutely continuous solutions of a certain nonlinear hyperbolic partial differential equation, 677.
Tucker, H. G. A generalization of the Glivenko-Cantelli theorem, 474.

Ulrick, Bruno. See Greenspan, Donald.
Utz, W. R. See Cunkle, C. H.

Vanderslice, B. R. and Hall, D. W. Chromatic polynomials, 852.
Vandrey, J. F. On the numerical evaluation of improper integrals, 26.
Varga, R. S. Orderings of the successive overrelaxation method, 208.
——See Birkhoff, Garrett.
Vaught, R. L. Universal relational systems for elementary classes and types, 671; Prime models and saturated models, 780; Homogeneous universal models of complete theories, 775.

Wagner, A. See Ostrom, T. G.
Walker, E. A. Subdirect sums and infinite Abelian groups. Preliminary report, 196.
——See Boyer, D. L.
Wall, D. W. Socles and UMFR algebras, 77; A characterization of generalized uniserial algebras, 316; Cartan invariants of UMFR algebras, 494; Basic algebras of UMFR algebras, 841.
Wallace, A. H. Analytic equivalence of curves, 205; Sheets of real analytic varieties, 800.
Walsh, J. L. Note on least-square approximation to an analytic function by polynomials, as measured by a surface integral, 668; Solution of the Dirichlet problem for the ellipse by interpolating harmonic polynomials, 773.
Walsh, J. L. and Landau, H. J. On canonical conformal maps of multiply connected domains. II, 154.
Walsh, J. L. and Russell, H. G. Integrated continuity conditions and degree of approximation by polynomials or by bounded analytic functions, 348.
Walsh, J. L. and Sinclair, Annette. On the degree of convergence of sequences of extremal functions, 487.
Walter, J. H. Structure of cleft rings. I, 678.
Wang, H. C. Invariant connections over a principal fibre bundle, 300; Compact transformation groups of n-sphere with an (n - 1)-dimensional orbit, 668.
——See Boothby, W. M.
Ward, L. E., Jr. On dendritic sets, 85; Fixed point theorems for arcwise connected spaces, 375.
Ward, Morgan, Arithmetical properties of Bernoulli polynomials, 230; A diophantine problem associated with the homogeneous product sum, 380.
Warner, S. L. The topology of compact convergence on continuous function spaces, 85; Characters of Cartesian products of rings, 509; Compact principal domains, 689.
Wasel, A. D. On the variation of parameters, 82.
Wasow, W. R. Solution of nonlinear differential equations with a parameter by asymptotic series, 204.
Waterman, Daniel. On the convergence of

the Laplace-Stieltjes transform, 590.
——See Goffman, Casper.
Weinberg, Louis and Slepian, Paul. Positive real matrices, 356.
Weinberger, H. F. Lower bounds for higher eigenvalues by finite difference meth ods, 34; Mean square approximation in symmetric hyperbolic systems by a finite difference method, 170.
——See Payne, L. E.
Weinstein, Alexander. On convex functions, 73; On the heat equation with subharmonic initial data, 322.
Weinzweig, A. I. The fundamental group of the union of spaces, 594.
Weiss, Mary. On the law of the iterated logarithm, 588; On a problem of Littlewood, 588; On the law of the iterated logarithm for uniformly bounded orthonormal systems, 588.
Weiss, Paul. The initial value problem of nth order linear differential and difference equations with constant coefficients, 316.
Wells, C. P. See Alavi, Yousef.
Wells, J. H. Concerning the Hausdorff inclusion problem, 374.
Wend, D. V. V. Some nonoscillation theorems on the real axis for complex differential equations, 672.
——See Coles, W. J.
Wendel, J. G. Order statistics of partial sums, 693.
Wendroff, Burton. On centered difference equations for hyperbolic systems, 462.
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Whaples, George. Quadratic lattices over local fields, 602.
White, J. S. A moment analogue to Spitzer's theorem, 483.
Whyburn, G. T. Topological characterization of the Sierpinski curve, 160; On convergence of mappings, 161; Uniform convergence for monotone mappings, 161; Quasi-open mappings, 161; On the invariance of openness, 455; Compactness of certain mappings, 456.
Widder, D. V. See Rosenbloom, P. C.
Widom, Harold. An extremum problem, 45; Inversion of Toeplitz matrices. II, 669.
——See Pollard, Harry; Spitzer, Frank.
Wiegmann, N. A. The structure of unitary and orthogonal quaternion matrices, 346.
Wilansky, Albert and Zeller, Karl. FH spaces and intersection of FK spaces, 478.
Wilcox, C. H. Spherical means and radiation conditions, 816.
Willcox, A. B. On honogeneous algebras of functions, 588.
Williams, R. F. Lebesgue area for compact two-dimensional surfaces, 301; A general inequality for a Lebesgue type area, 852.
Williamson, R. E. See Snell, J. L.
Willoughby, R. A. and Chen, T. C. Computation of Eigenvalues and vectors of Hermitean matrices, 153.
Wilson, J. C. Periodic solutions to an equation with rational coefficients, 171.
Wilson, R. L. A prediction problem in real-time, 199.
Wisner, R. J. See Beaumont, R. A.; Greenleaf, Newcomb.
Wolf, František. Extension of the spectral decomposition theorem, 68.
Wolfe, Philip. Simplex method for quadratic programming. II, 358; Programming with nonlinear constraints. Preliminary report, 508.
Wolfowitz, Jacob. The maximum achievable length of an error correcting code, 336; Strong converse of the coding theorem for discrete finite-memory channels, 669.
Woll, J. W., Jr. Subordination of infinitely divisible processes, 852.
Woods, A. C. On a problem of Mahler's, 509.
Wright, F. M. Some extensions of results for C-fractions of a set Γ, 92.
Wright, J. B. See Büchi, J. R; Elgot, C. C.
Wyler, Oswald. Damped free vibrations with a finite number of oscillations, 211; Ideal points and order axioms in an incidence geometry, 342.

Yamabe, Hidehiko. A unique continuation theorem of a parabolic equation, 164; A deformation theorem on compact manifolds, 351.
Yamada, Miyuki. See Kimura, Naoki.
Yamamuro, Sadayuki. On Nakano spaces, 42.
Yang, C. T. p-adic transformation groups, 508.
Yeh, G. C. K. and Martinek, Johann. Disturbance of a many-dimensional field

satisfying the Helmholtz equation due to the presence of a hyperplane boundary, 461.
Yen, Ti. See Silverman, R. J.
Young, G. S. Generalizations of the Liouville theorem to higher dimensions, 208; A generalization of Bagemihl's theorem on ambiguous points, 301; Reversal of Peano continua by analytic maps, 483; A reflection theorem for light interior functions, 842.
——See Jones, F. B.
Young, L. C. Approximate radius and approximate diameter, 58.
Younglove, J. N. Concerning dense metric subspaces of certain nonmetric spaces, 669.

Zadunaisky, P. E. On the group iteration and group overrelaxation methods of solving a system of linear equations, 52.
Zelinsky, Daniel. See Rosenberg, Alex.
Zeller, Karl. See Brown, Leon; Wilansky, Albert.
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Zuckerman, H. S. See Hewitt, Edwin.

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